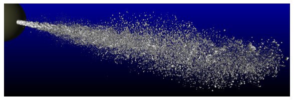
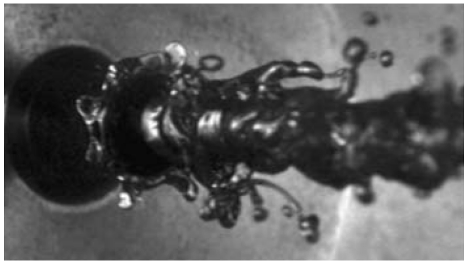
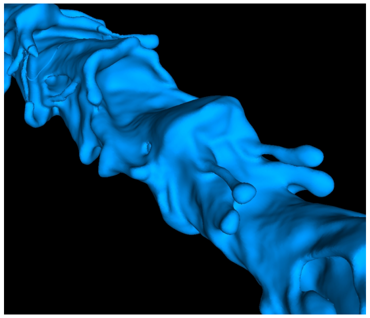


MCEN 6228
Multiphase Flows

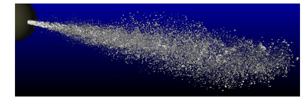


Olivier Desjardins



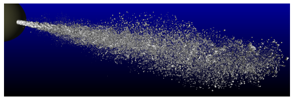
Department of Mechanical Engineering

Fall 2010



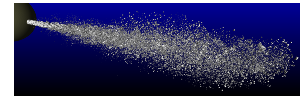
Overview

- Class: Tue, Th, 11:00am – 12:15pm
- Office hours: Walk-in or by appointment
- Books:
  - C. Crowe, M. Sommerfeld, Y. Tsuji: Multiphase Flows with Droplets and Particles
  - Multiphase Flow Handbook, C. T. Crowe
  - W. A. Sirignano: Fluid Dynamics and Transport of Droplets and Sprays
  - Atomization and Sprays, A. H. Lefebvre



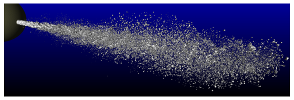
Coursework

- Homework: No homework
- Mid term quiz: No midterm
- Final exam: No final



Class Projects

- Choose topic by end of next week
  - 2-page proposal due 09/02/2010 in class
- Prepare progress report (seminar and write-up) that summarizes the topic and which can be used by other students
  - 10 minutes presentations given on Oct. 12 – 14
  - 5-page write-up
- Final report (seminar and write-up) during the last two weeks of class
  - 15 minutes presentations given on Nov. 30 – Dec. 9
  - 10-page write-up
- Presentations graded by your peers



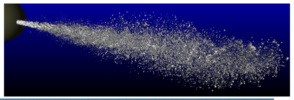
Objectives & Pre-requisites

Investigate a Variety of Complex Multiphase Flows

- Focus on global flow properties
- Provide theoretical framework
- Discuss state of the art modeling strategies
  - Sometimes theoretical
  - More often computational models

Pre-reqs

- Navier-Stokes, anyone?
- In general, mass, momentum, energy conservation
- Better not be afraid of pdes
- Some familiarity with numerical methods



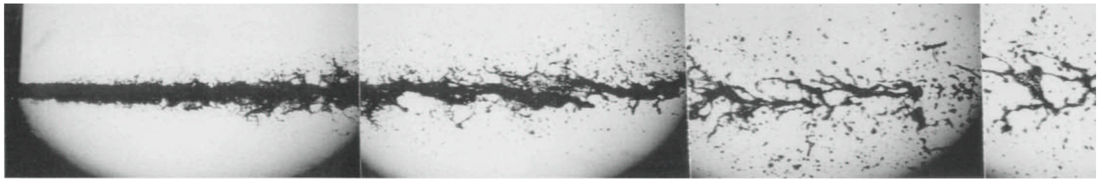
Course Topic

wave breaking

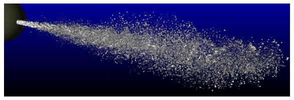
Variety of Multi-Phase Flows

- Spray combustion
- Bubbles
- Splashing water
- Ocean, breaking waves
- Solid particles
- Inkjet printer

truck tire splash

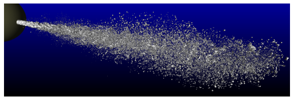


LOX+GH2 cold jet (Mayer et al. 01)

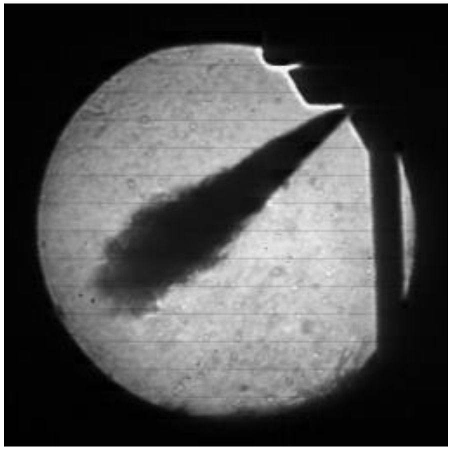


Some Applications

- Combustion of liquid fuels
  - Aircraft engines
  - Diesel engines
  - Fluidized bed coal combustion
- Interfacial flows
- Particle laden flows
- Condensation and evaporation in clouds
- Bubble columns



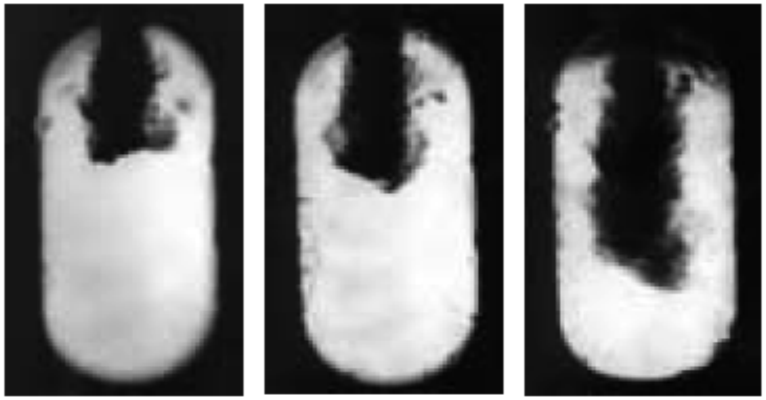
Issues

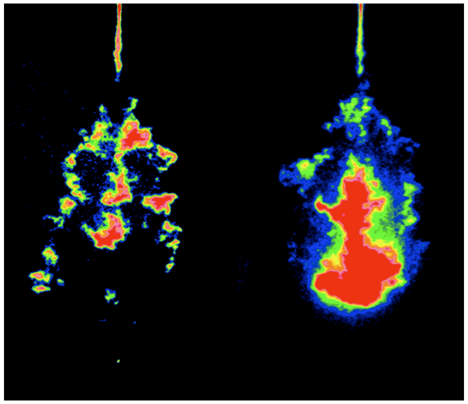


Diesel injection

- Primary breakup
  - Instabilities
- Secondary breakup
  - Droplets into smaller droplets
- Evaporation, condensation
- Coalescence or particle interactions
- Momentum/energy/mass exchange of different phases

Diesel injection

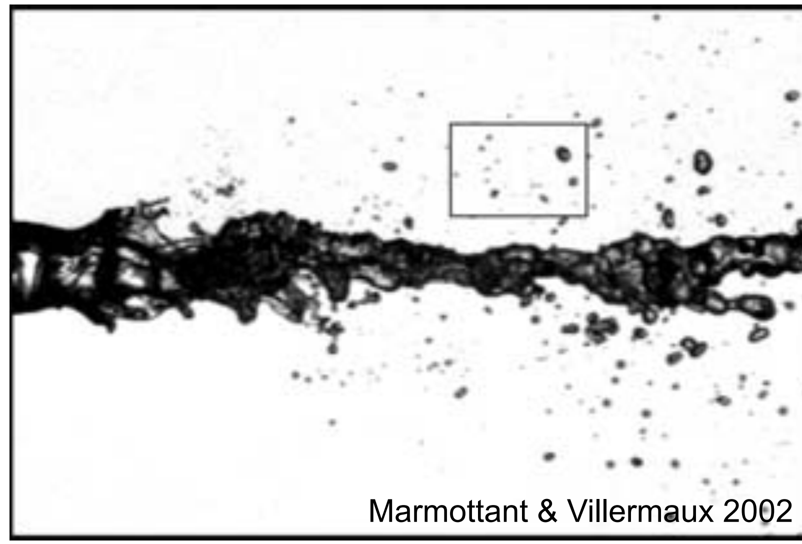




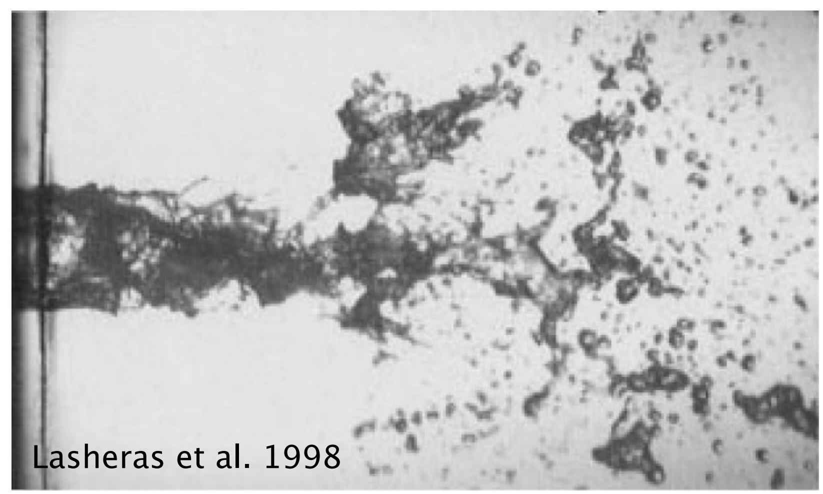
Example

- Breakup of a liquid jet
- Liquid core into large drops
- Large drops into small drops
- Or direct disintegration

coaxial atomization



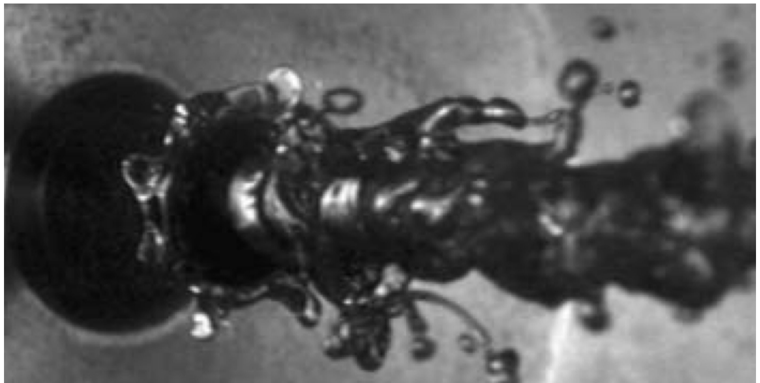
Marmottant & Villermaux 2002



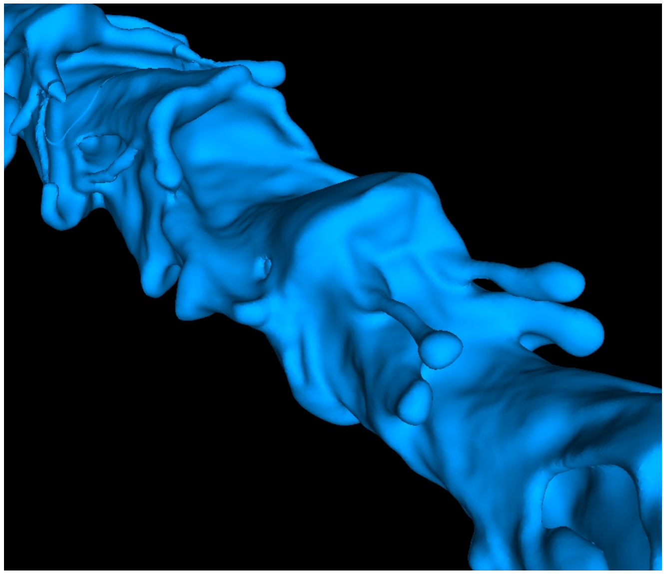
Lasheras et al. 1998

Example

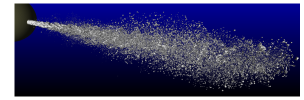
- Formation of large drops from experiment and simulation



Marmottant & Villermaux 2004



Herrmann et al. 2006



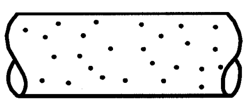
Multi-Phase Flow: Classification

Liquid/Gas
Drops
Bubbles
Foam
Ocean surface (moving)

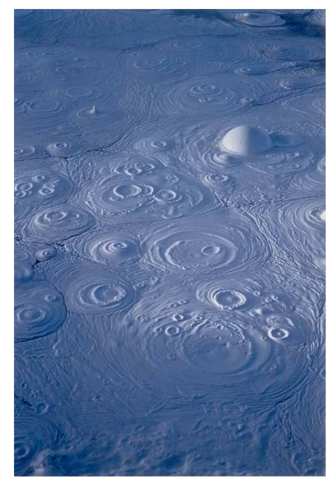
Gas

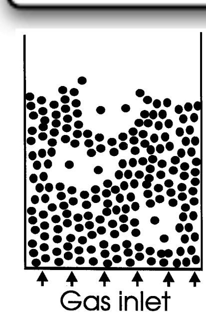
Liquid

Solid/Gas
Particle laden flow
Fluidized bed

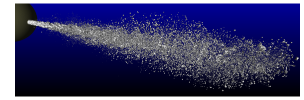


Solid/Liquid
Particle laden flow
Slurry, mud



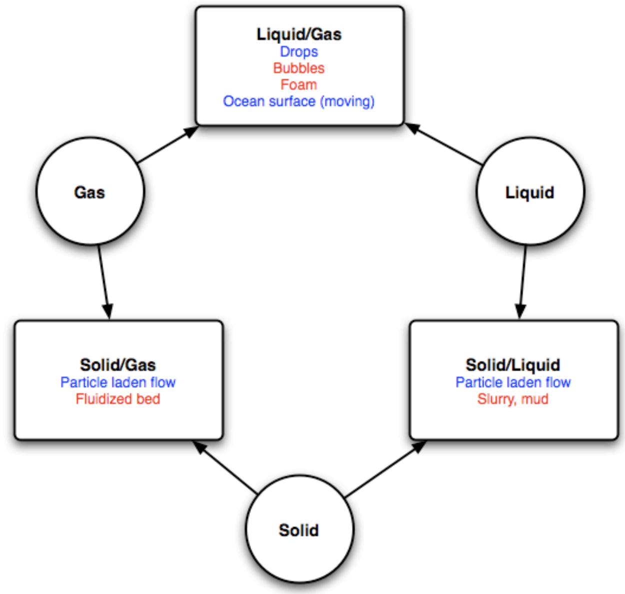


Solid

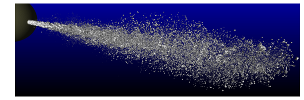


Multi-Phase Flow: Classification

- Multiphase flows with assumed phase-interface topology
  - Particle-laden flows, sprays
  - Dispersed or dense?
  - Lagrangian or Eulerian description?
  - Heat and mass transfer?

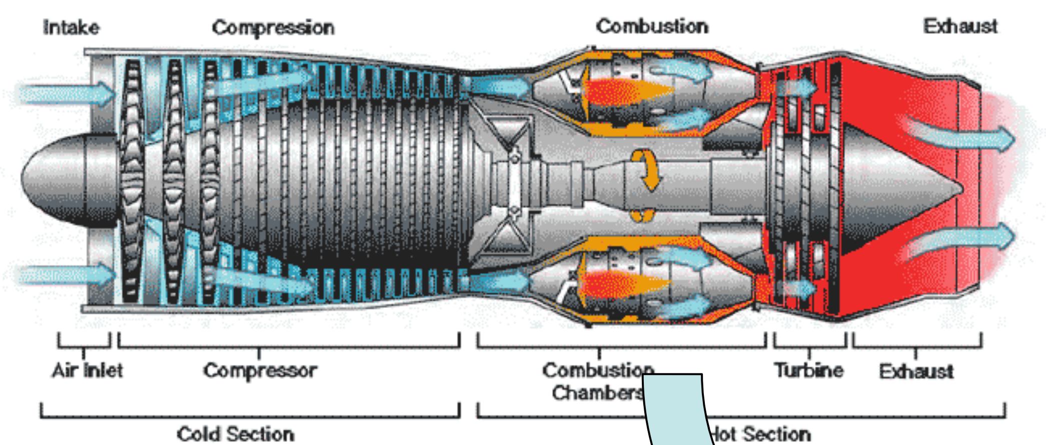


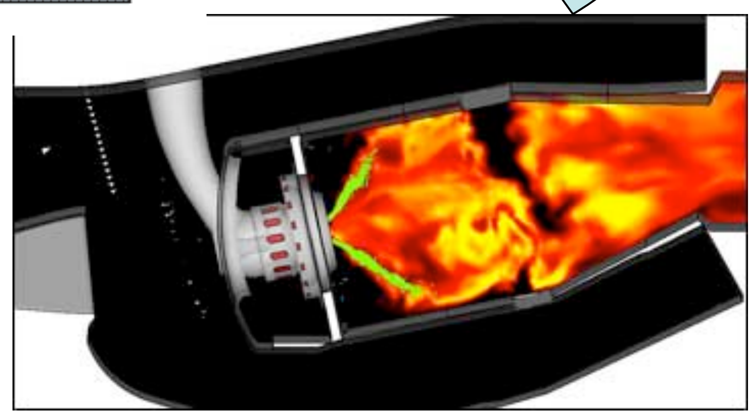
- Multiphase flows with resolved phase-interface topology
  - Gas-liquid flows
  - Various models
  - Heat and mass transfer



Gas turbine engine

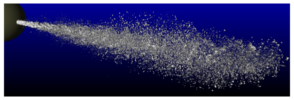
- Example of gas turbine combustion chamber
  - Involves a range of multiphase flows





Gas turbine engine

- Example of gas turbine combustion chamber
  - Involves a range of multiphase flows
    - Fuel injection, primary atomization
    - Secondary atomization
    - Droplet coalescence
    - Turbulent spray dispersion
    - Spray-wall interactions
    - Spray evaporation
    - Spray combustion
    - Others…



Dispersed two-phase flows

Definition of Number Density

- Consider gas-phase density
- Define local gas density as

$$\rho = \lim_{\delta V \to \varepsilon^3} \frac{\delta m}{\delta V}$$

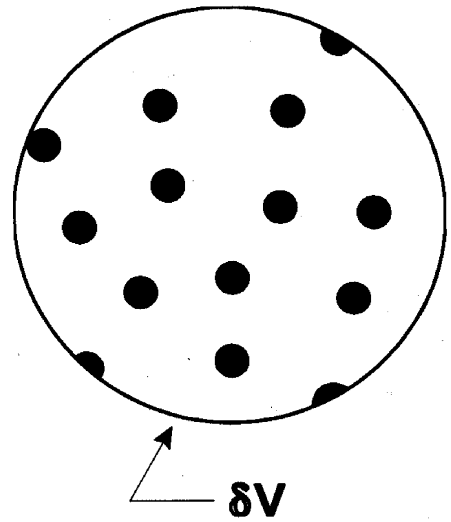
- Mean free path: ~ 100 nm

→ ε >> 100 nm for statistical average

Number Density

- Number density

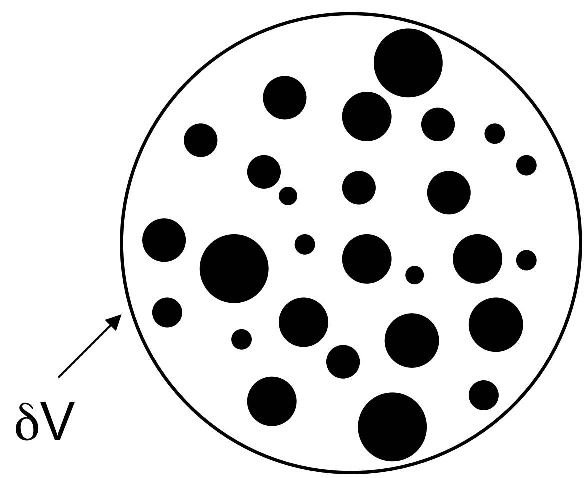
$$n = \lim_{\delta V \to \delta V^{\circ}} \frac{\delta N}{\delta V} \qquad \left[\frac{\#\text{particles}}{\text{volume}}\right]$$

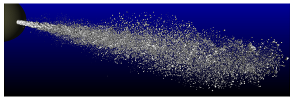


- Implications
  1. Averaging volume has to be much larger than droplet spacing
  2. Evaporating droplets
     - Not all particles are same size

⇨ Particle size distribution

Statistical model

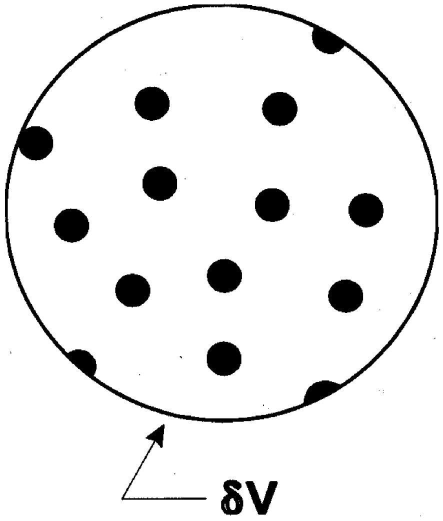




Volume fraction

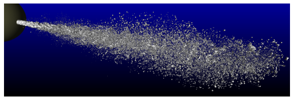
- Look at particle volume per unit volume

$$\alpha_p = \lim_{\delta V \to \delta V^o} \frac{\delta V_p}{\delta V} \qquad \left[\frac{\text{volume particle}}{\text{volume}}\right]$$



- Can define interparticle spacing
  - For a cubic arrangement

$$L/d_p = \left(\frac{\pi}{6\alpha_p}\right)^{1/3}$$



Particle Size Distribution Function

PSDF: Probability Density Function of particle diameter for ensemble of particles

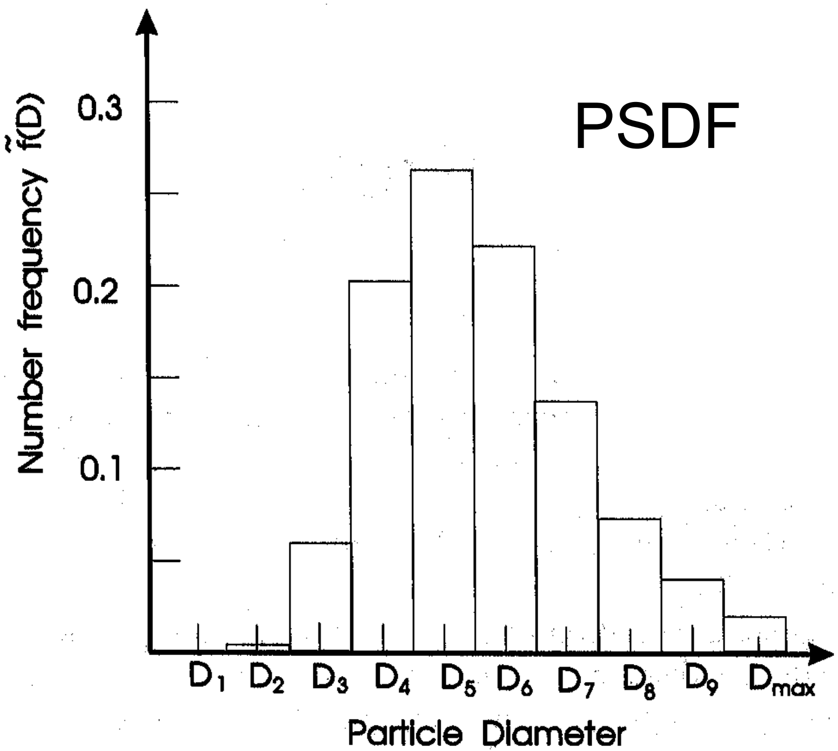
Discrete Form

- Discrete PSDF: $\tilde{f}_n(D_i)$
- Normalization:

$$\sum_i \tilde{f}_n(D_i) = 1$$

- Note:
  - PSDF sometimes not normalized, then

$$\sum_i \tilde{g}_n(D_i) = n$$



Particle Size Distribution Function

- Mean

$$\overline{D}_n = \sum_i D_i \tilde{f}_n(D_i)$$

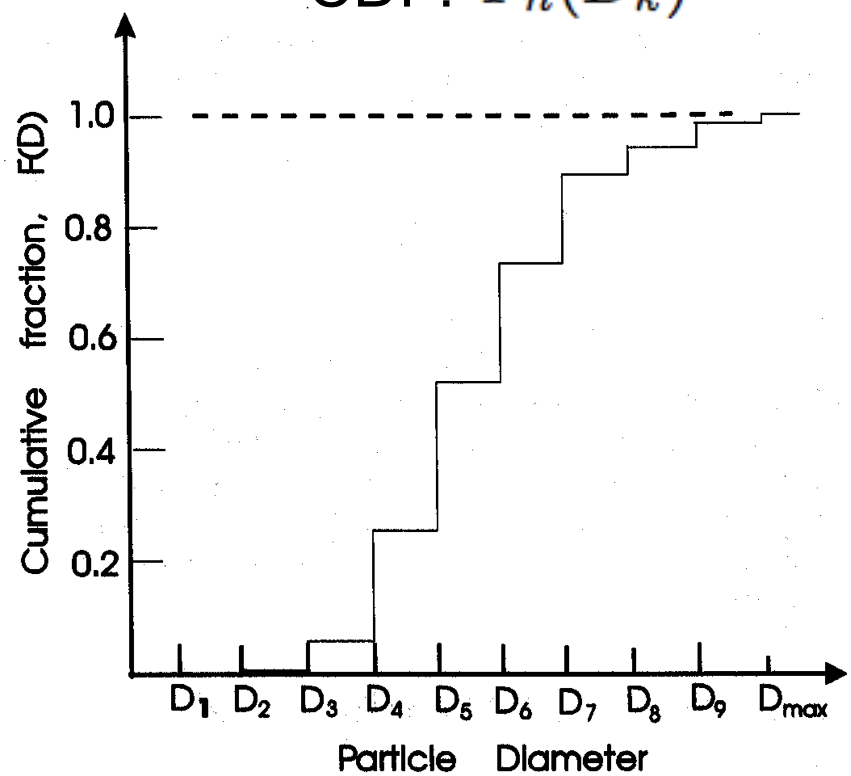
- Variance

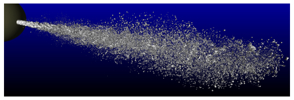
$$\sigma_n^2 = \sum_i (D_i - \overline{D}_n)^2 \tilde{f}_n(D_i)$$

- Cumulative Distribution Function

$$\tilde{F}_n(D_k) = \sum_{i=1}^{k} \tilde{f}_n(D_i)$$

CDF: $\tilde{F}_n(D_k)$





Particle Size Distribution Function

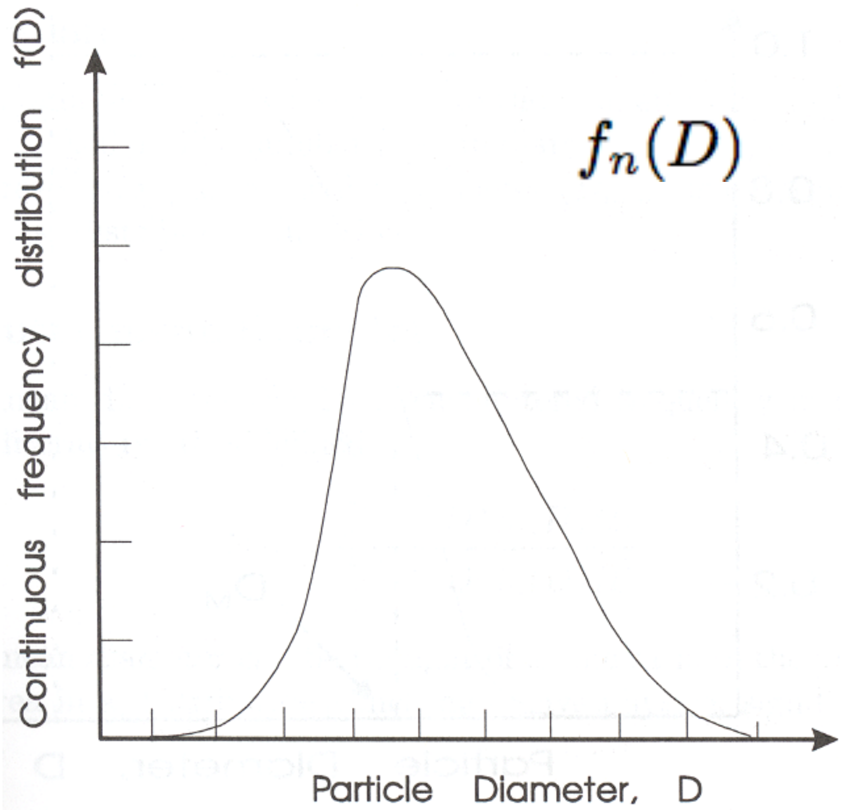
Continuous Form

- Continuous PSDF: $f_n(D)$

- Normalization:

$$\int_0^\infty f_n(D)dD = 1$$

- Straightforward definitions of moments and CDF



Characteristic Diameters

- Other possible definitions of characteristic diameters
- Example
  - Median

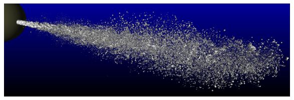
$$\int_0^{D_M} f_n(D)dD = 0.5$$

  - Ratio of moments

$$D_{mk} = \left(\frac{\int_0^\infty D^m f_n(D)dD}{\int_0^\infty D^k f_n(D)dD} \right)^{1/(m-k)}$$

  - Special form: Sauter Mean Diameter (SMD)
    - Corresponds to total volume over total surface

$$D_{32} = \left(\frac{\int_0^\infty D^3 f_n(D)dD}{\int_0^\infty D^2 f_n(D)dD} \right)$$

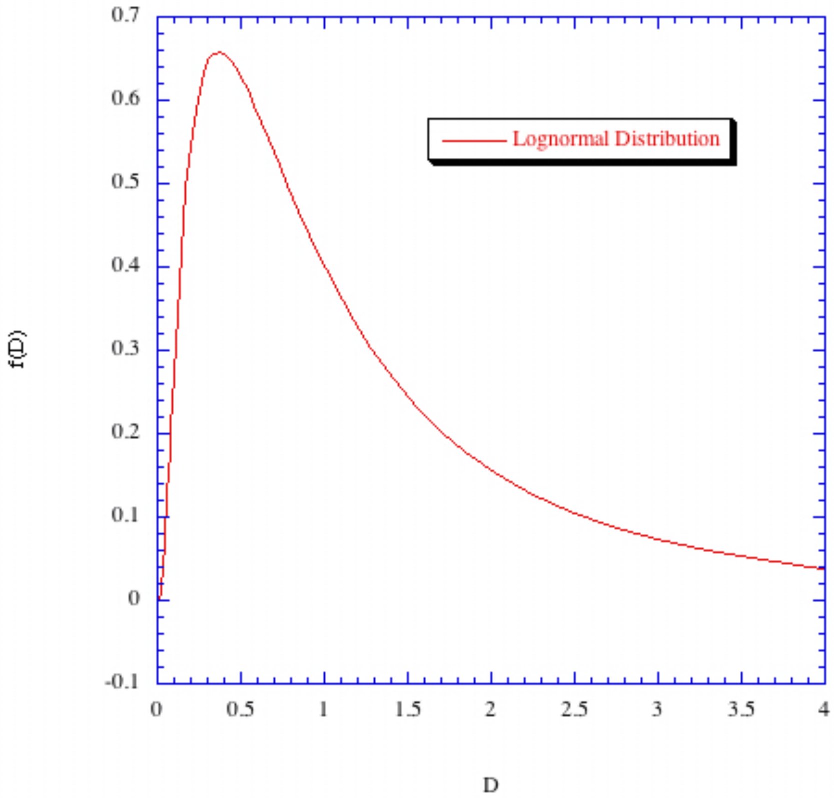


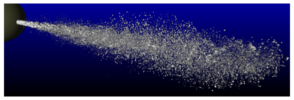
Frequently Used PSDFs

- Log normal

$$f(D) = \frac{1}{D\sqrt{2\pi}\sigma_0} \exp\left(-\frac{1}{2}\left(\frac{\ln D - \ln D_{nM}}{\sigma_0}\right)^2\right)$$

- σ_0 is variance of log of *D*

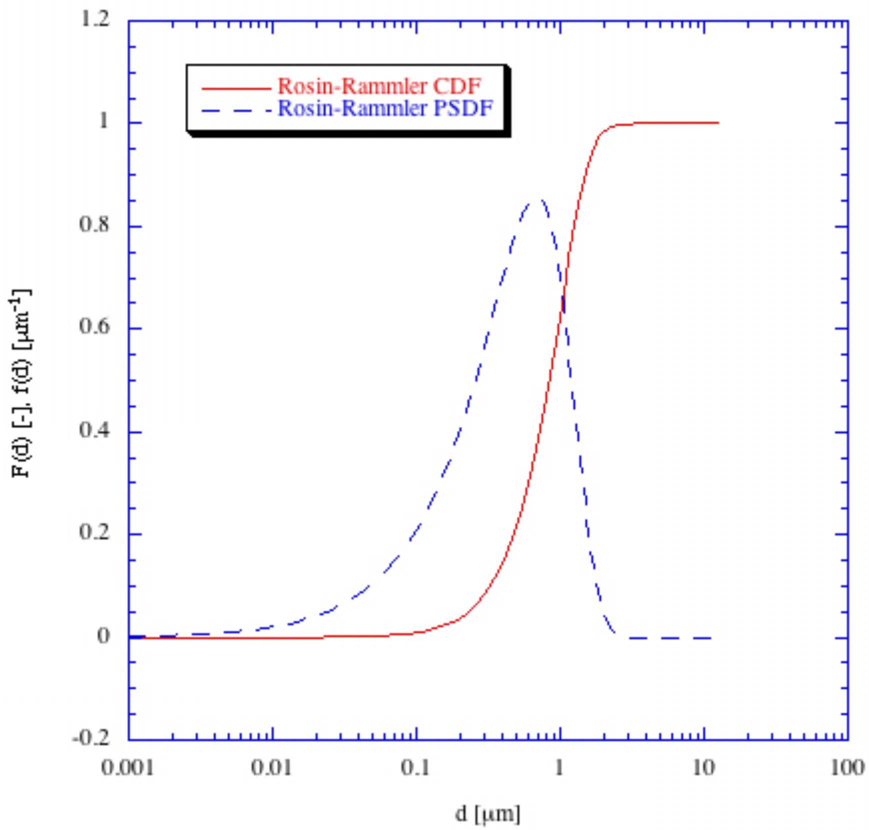


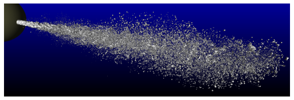


Frequently Used PSDFs

- Rosin-Rammler Distribution
  - Often used to describe droplet sizes in sprays
  - Defined by mass density function with empirical constants δ and *n*

$$F_m(D) = 1 - \exp\left[-\left(\frac{D}{\delta}\right)^n\right]$$





Dispersed Phase Flows

Volume Fraction

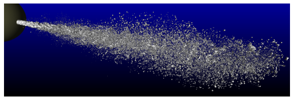
- Dispersed phase volume fraction

$$\alpha_d = \lim_{\delta V \to \delta V^o} \frac{\delta V_d}{\delta V}$$

- Continuous phase volume fraction, aka void fraction

$$\alpha_c = \lim_{\delta V \to \delta V^o} \frac{\delta V_c}{\delta V} = 1 - \alpha_d$$

- δV^o has to be large enough to ensure converged statistics



Particle Spacing *L*

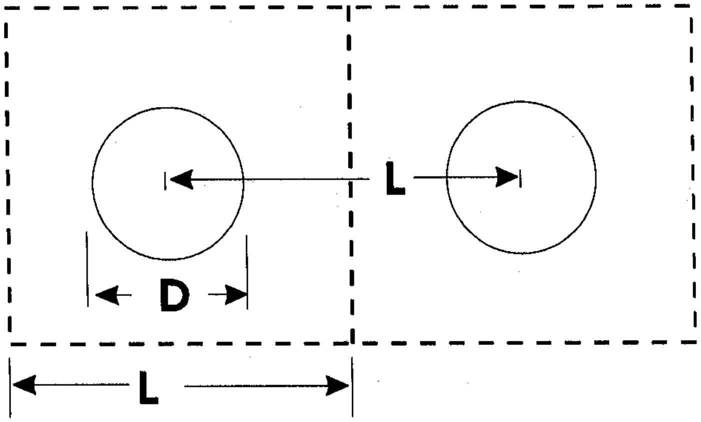
When can particle/particle interactions be neglected?

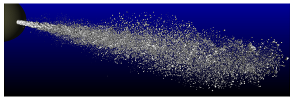
- PPI negligible if $L/D > 10$
- Dispersed phase

$$\alpha_d = \frac{\pi D^3}{6L^3}$$

- PPI negligible if $\alpha_d < 5 \cdot 10^{-4}$
- Seems awfully small!
- How much is this in particle mass ratio?

Two particles with distance *L*





Particle Spacing *L*

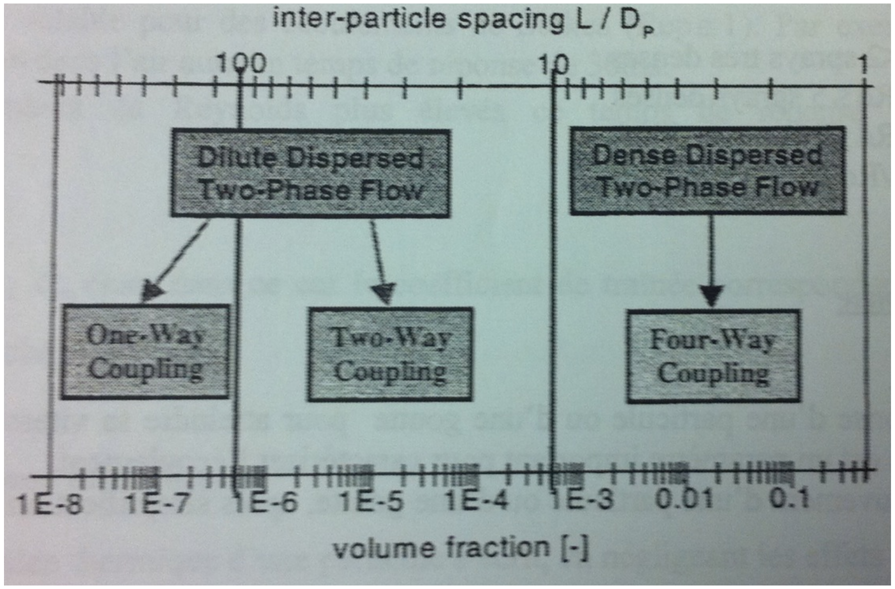
- Particle mass ratio

$$C = \frac{m_d}{\rho_c(L^3 - \pi/6D^3)} = \frac{\pi}{6}\frac{\rho_d/\rho_c}{(L/D)^3 - \pi/6}$$

- From L/D = 10 follows for water droplets in air

$$C \approx 0.5$$

Classification



- One-way coupling
  - Gas affects particles through drag, evaporation
- Two-way coupling
  - Gas affects particles through drag, evaporation
  - Particles affect gas through drag, evaporation
- Four-way coupling
  - Gas affects particles through drag, evaporation
  - Particles affect gas through drag, evaporation
  - Particles affect each other through collisions, coalescence

Particle Spacing *L*

How large does δV^o have to be, if N particles are required for a statistical ensemble?

- Dispersed phase

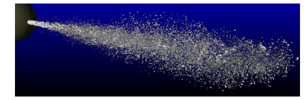
$$\delta V^o = N \cdot L^3 = N \left(\frac{L}{D}\right)^3 D^3$$

- For 10 μm particles, *L/D* = 10, and the requirement to have 1000 particles in ensemble follows

$$\delta V^o = 1\,\mathrm{mm}^3$$

- For 0.1 mm particles

$$\delta V^o = 1\,\mathrm{cm}^3$$



Time Scale Estimates

Particle Response Time

- Particle equation of motion

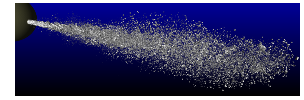
$$m \frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}$$

- Drag force

$$\boldsymbol{F} = c_D \frac{\rho_c}{2} \frac{\pi D^2}{4} |\boldsymbol{u} - \boldsymbol{v}| (\boldsymbol{u} - \boldsymbol{v})$$

- gives

$$m \frac{d\boldsymbol{v}}{dt} = c_D \frac{\rho_c}{2} \frac{\pi D^2}{4} |\boldsymbol{u} - \boldsymbol{v}| (\boldsymbol{u} - \boldsymbol{v})$$



Time Scale Estimates

- Introducing the Reynolds number as

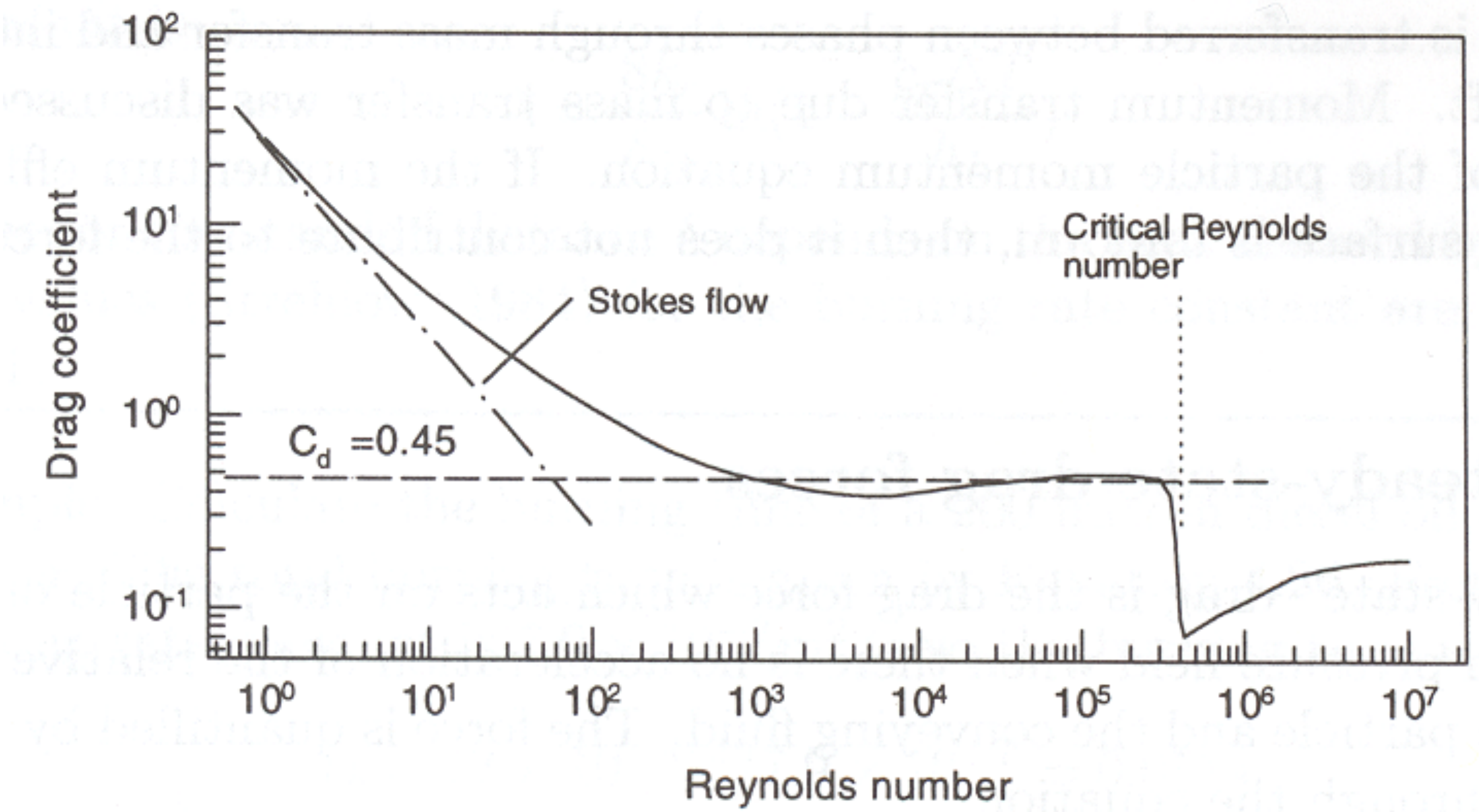
$$\mathrm{Re}_{\mathrm{rel}} = \frac{\rho_c D |\boldsymbol{u} - \boldsymbol{v}|}{\mu_c}$$

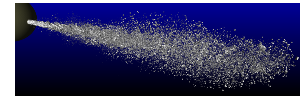
- gives

$$\frac{d\boldsymbol{v}}{dt} = \frac{18}{24} \frac{c_D \mathrm{Re}_{\mathrm{rel}} \mu_c}{D^2 \rho_d} (\boldsymbol{u} - \boldsymbol{v})$$

Time Scale Estimates

Drag coefficient of a sphere





Time Scale Estimates

- Small Reynolds number: Stokes flow

$$c_D = \frac{24}{\mathrm{Re}_{\mathrm{rel}}}$$

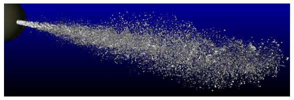
- Leads to

$$\frac{d\boldsymbol{v}}{dt} = \frac{1}{\tau_v}(\boldsymbol{u} - \boldsymbol{v})$$

- With

$$\tau_v = \frac{D^2 \rho_d}{18\mu_c} = \frac{D^2}{18\nu_c}\frac{\rho_d}{\rho_c}$$

- τ_v is characteristic time to reach equilibrium of velocities

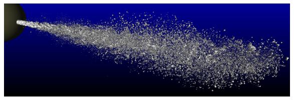


Time Scale Estimates

Particle Response Time

- For water in air and D = 0.1mm

$$\tau_v \approx 50\,\mathrm{ms}$$



Time Scale Estimates

- Large Reynolds number: $c_D \approx 0.5$
- Leads to

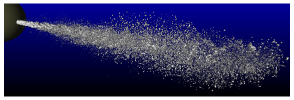
$$\frac{d\boldsymbol{v}}{dt} = \frac{1}{\tau_{vl}}(\boldsymbol{u} - \boldsymbol{v})$$

- With

$$\tau_{vl} = \frac{8}{3}\frac{D}{|\boldsymbol{u} - \boldsymbol{v}|}\frac{\rho_d}{\rho_c}$$

- For water in air, D = 1mm, v_{rel} = 10m/s, which results in Re = 1000

$$\tau_{vl} = 0.3\,\mathrm{s}$$



Time Scale Estimates

Particle Response Time

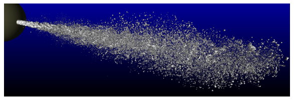
- Velocity or momentum difference can be neglected if the shortest flow time scale is much larger than particle response time

$$\tau_F \gg \tau_v$$

- Example: LES
  - For Δ = 1mm, U = 100m/s follows

$$D \approx 1.4\,\mu\text{m}$$

  - For this case, relative velocity can be neglected for particles smaller than D = 1μm



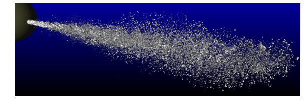
Time Scale Estimates

Stokes Number

- Stokes number describes ratio of particle time scale to flow time scale

$$\mathrm{St} = \frac{\tau_v}{\tau_F}$$

- Effect illustrated by constant time lag solution



Time Scale Estimates

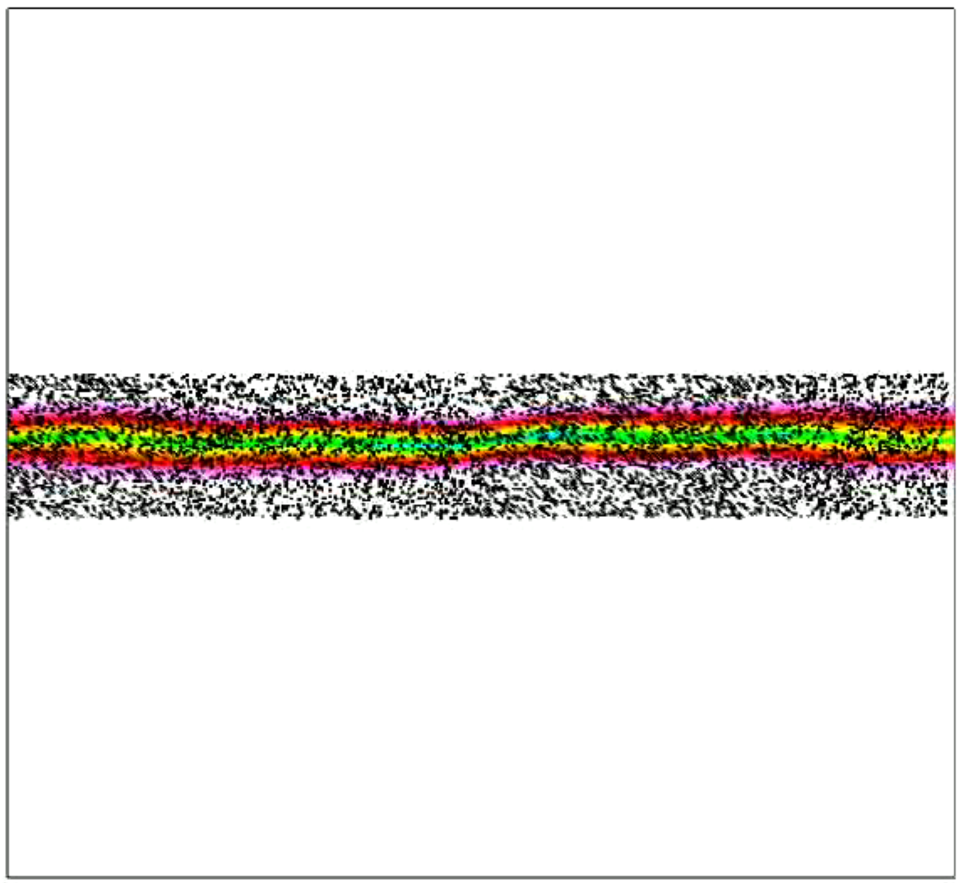
Constant Time Lag Solution

$$\frac{v}{u} = \frac{1}{1 + \mathrm{St}}$$

- St = 0 (very small particles): v = u
- St -> ∞ (very large particles): v = 0
- St ≈ 1: strong interaction of u and v

Time Scale Estimates

Effect of Stokes Number



Time Scale Estimates

Thermal Response Time

- Heat flux

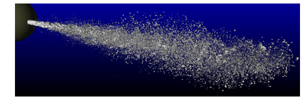
$$\dot{q}'' = \lambda_c \left.\frac{dT_G}{dr}\right|_{\mathrm{surface}} = k(T_\infty - T_d)$$

- Define non-dimensional heat transfer coefficient *k*

$$\mathrm{Nu} = \frac{kD}{\lambda_c}$$

- Conductive heat transfer

$$\dot{Q} = \dot{q}''\pi D^2 = \mathrm{Nu}\pi D\lambda_c(T_\infty - T_d)$$



Time Scale Estimates

- Droplet energy equation

$$m c_{pd} \frac{dT_d}{dt} = \dot{Q}$$

- gives

$$\frac{dT_d}{dt} = \frac{6 \mathrm{Nu} \lambda_c}{\rho_d D^2 c_{pd}} (T_\infty - T_d)$$

Time Scale Estimates

- Small Reynolds number: Nu = 2 gives

$$\tau_T = \frac{\rho_d c_{pd} D^2}{12\lambda_c}$$

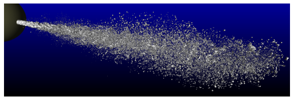
- With Prandtl number defined as

$$\mathrm{Pr} = \frac{\mu_c c_{pc}}{\lambda_c}$$

- Time scale ratio

$$\frac{\tau_v}{\tau_T} = \frac{2}{3}\frac{1}{\mathrm{Pr}}\frac{c_{pc}}{c_{pd}}$$

- Conclusion: Thermal equilibrium typically slower than velocity



Collision Time Scale

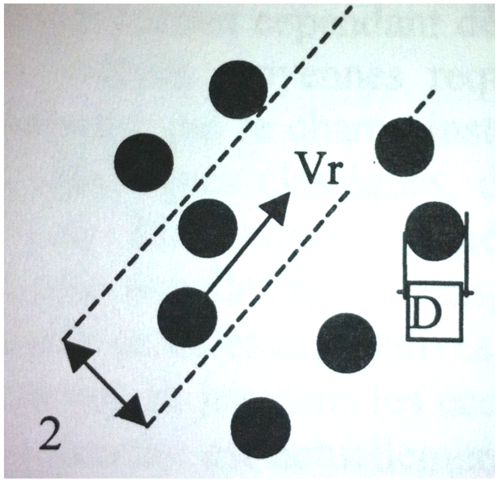
- Other way of characterizing dense dispersed two-phase flow

$$\tau_v/\tau_c < 1 \qquad \text{DILUTE}$$

$$\tau_v/\tau_c > 1 \qquad \text{DENSE}$$

- Collision timescale

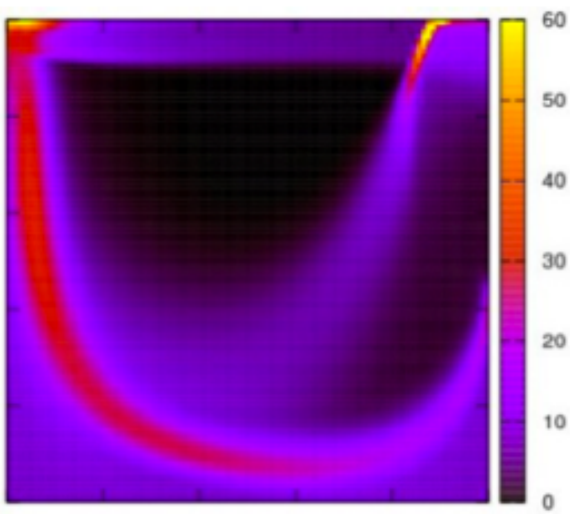
$$\tau_c = \frac{1}{n\pi D^2 v_r}$$

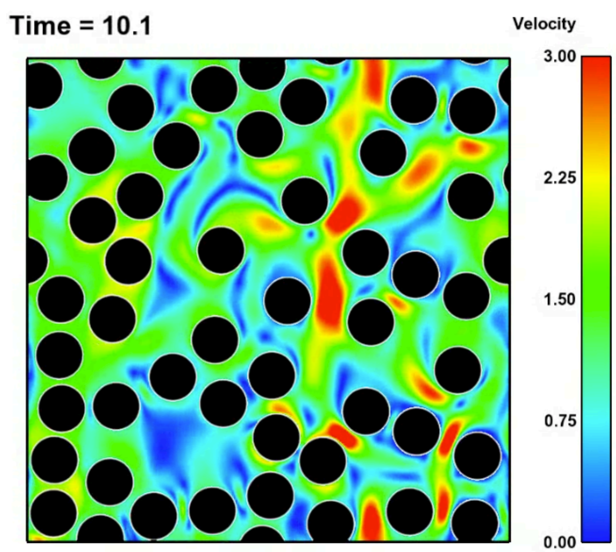


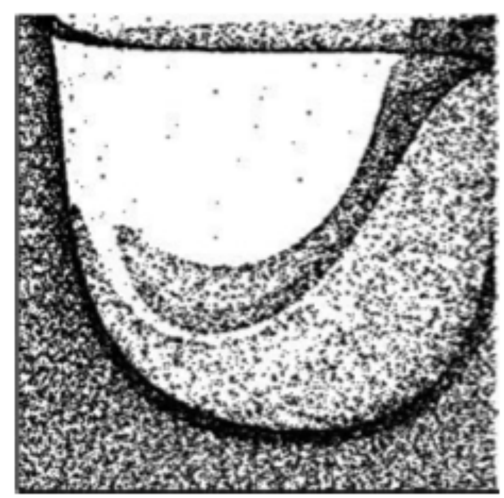
- How should we define the relative velocity?

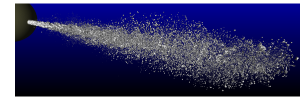
Governing Equations

1. Flow equations locally considering ensembles of particles (Eulerian representation)
2. Flow equation describing flow locally around a single particle and inside droplets (DNS)
3. Lagrangian particle equations (Lagrangian tracking)









Particle Equations

Lagrangian description of single particle or droplet

- Position

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

- Momentum

$$\frac{d\boldsymbol{v}}{dt} = \frac{f_1}{\tau_v}(\boldsymbol{u} - \boldsymbol{v}) + \boldsymbol{g}$$

- f_1 is factor describing departure from Stokes flow and Stefan flow (blowing effect)

Particle Equations

- Energy

$$c_{pd} m \frac{dT_d}{dt} = \dot{Q} + (h_c - h_d)\dot{m}$$

$$\frac{dT_d}{dt} = \frac{f_2 \mathrm{Nu}}{3\tau_v \mathrm{Pr}} \frac{c_{pc}}{c_{pd}} (T_\infty - T_d) + \frac{L_v}{c_{pd}} \frac{\dot{m}}{m}$$

- f_2 is factor correcting for evaporation effect on heat transfer (Stefan flow)
- Nu here is the corrected value considering convection

Particle Equations

- Mass

$$\frac{dm}{dt} = \rho_d \dot{r} \pi D^2 = -k_m \pi D^2 H_m$$

  - H_m is driving potential for mass transfer (like ΔT for energy)
  - k_m is mass transport coefficient

- With Sherwood and Schmidt numbers defined as

$$\mathrm{Sh} = \frac{k_m D}{\rho_c D_{cv}} \quad \text{and} \quad \mathrm{Sc} = \frac{\mu_c}{\rho_c D_{cv}}$$

- Follows

$$\frac{dm}{dt} = -\frac{\mathrm{Sh}}{3\mathrm{Sc}} \frac{m}{\tau_v} H_M$$

Particle Equations

- Which drop has a higher *dm/dt*, large or small?

$$\frac{dm}{dt} = -\mathrm{Sh}\rho_c D_{cv} \pi D H_M$$

- Which has higher *dm/dt*, one large drop or two smaller drops with each half the mass of the large drop?
  - Small droplets

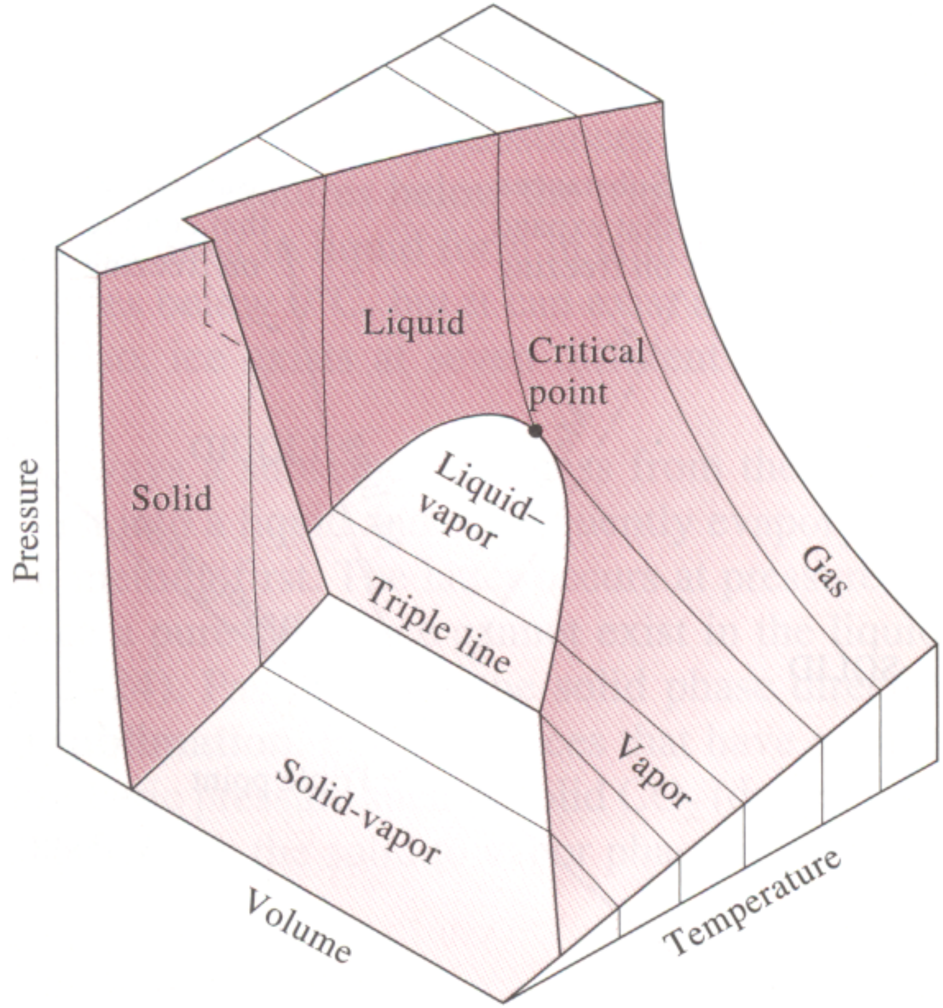
$$\frac{dm}{dt} = -2\mathrm{Sh}\rho_c D_{cv} \pi \frac{D_{1/2}}{D} D H_M$$

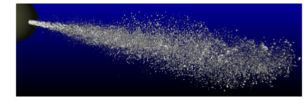
$$\frac{D_{1/2}}{D} = \left(\frac{m/2}{m}\right)^{1/3} = \frac{1}{2^{1/3}}$$

$$\frac{dm}{dt} = -2^{2/3}\mathrm{Sh}\rho_c D_{cv} \pi D H_M$$

Thermodynamics of Phase Change

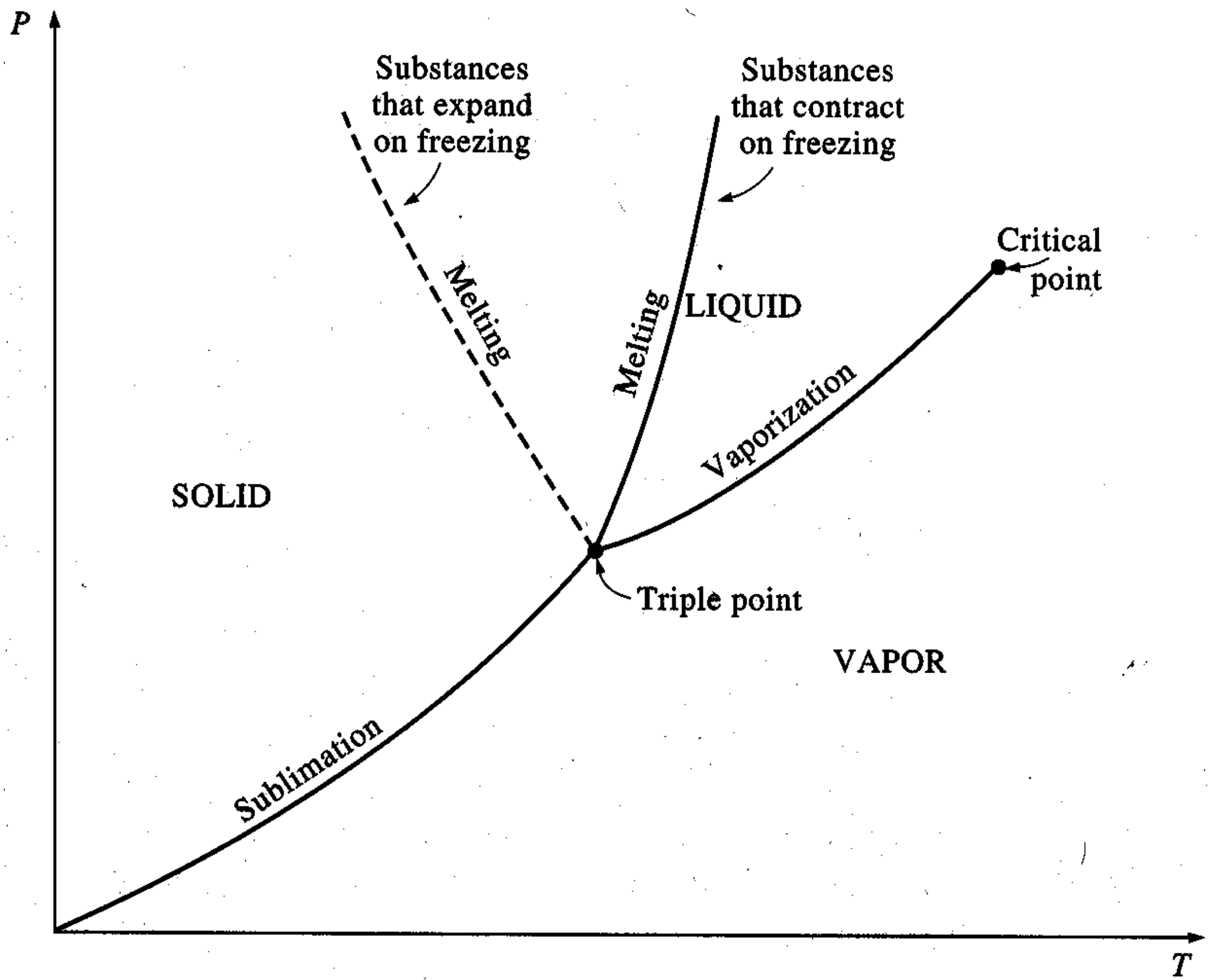
Phase Diagrams: p, v, T Surface





Thermodynamics of Phase Change

Phase Diagrams: Saturation Curve



Thermodynamics of Phase Change

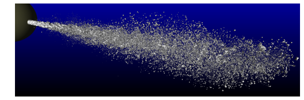
Vapor Mole Fraction at the Droplet Surface

- Clapeyron equation

$$\frac{dp}{dT} = \frac{h_{fg}}{T v_{fg}}$$

- Clausius-Clapeyron equation
  - Assumptions are
    - Equilibrium conditions
    - Ideal gas for vapor state
    - $v_l << v_g$
    - Small changes in L compared with reference state

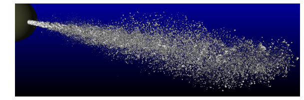
$$X_v = \frac{p_{\mathrm{atm}}}{p_G} \exp\left[\frac{L W_G}{\mathcal{R}} \left(\frac{1}{T_{b,\mathrm{atm}}} - \frac{1}{T} \right) \right]$$



Energy and Mass Transfer

Levels of Approximation

- Isolated droplets or droplet cloud
- Constant or time varying droplet temperature
- Constant or spatially varying droplet temperature
- Quiescent surrounding gas or convective flow
- Gas phase in steady state or time varying
- Influence of Stefan flow
- Phase equilibrium at the droplet surface



Energy and Mass Transfer

Simplest Model: D^2 Law for Evaporating Droplet

- Droplet mass change

$$\frac{dm}{dt} = -\mathrm{Sh}\rho_c D_{cv}\pi D H_M$$

$$D\frac{dD}{dt} = -\frac{2\mathrm{Sh}\rho_c D_{cv} H_M}{\rho_d}$$

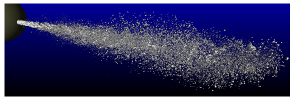
- Assume RHS constant leads to

$$\boxed{D^2 = D_0^2 - \lambda t} \text{ with } \quad \lambda = \frac{4\mathrm{Sh}\rho_c D_{cv} H_M}{\rho_d}$$

Energy and Mass Transfer

Spherically Symmetric Droplet Heating and Vaporization

- Assume steady gas-phase
- One-step global reaction
  - $\nu F + O \rightarrow (\nu + 1) P$
  - Reaction rate: ω



Energy and Mass Transfer

Gas-Phase Governing Equations

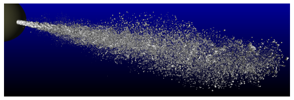
- Continuity

$$\frac{\partial}{\partial r}\left(\rho U r^2\right) = 0$$

$$\rho U r^2 = \frac{\dot{m}}{4\pi}$$

- Energy

$$\frac{\partial}{\partial r}\left(\rho U r^2 h\right) - \frac{\partial}{\partial r}\left(r^2 \rho D_c \frac{\partial T}{\partial r}\right) = -\rho r^2 Q \dot{\omega}$$



Energy and Mass Transfer

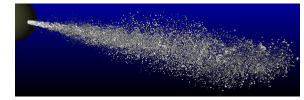
Gas-Phase Governing Equations

- Fuel

$$L(Y_F) = \frac{\partial}{\partial r}\left(\rho U r^2 Y_F\right) - \frac{\partial}{\partial r}\left(r^2 \rho D_c \frac{\partial Y_F}{\partial r}\right) = \rho r^2 \dot{\omega}$$

- Other species

$$L(Y_O) = \rho r^2 \frac{\dot{\omega}}{\nu}$$



Energy and Mass Transfer

Boundary Conditions

- At r -> ∞: Y_i, $T = Y_{i,\infty}$, T_∞
- At r = R:
  - Continuous temperature: $T_s = T_{l,s}$
  - X_F from Clausius-Clapeyron equation

$$Y_i = \frac{X_i W_i}{W} \quad \text{with} \quad W = \sum_j X_j W_j$$

  - Mass and energy balances from integration of species transport equations across interface

Energy and Mass Transfer

Flux Conditions

- Fuel

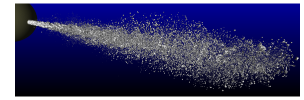
$$\frac{\dot{m}}{4\pi} Y_{F,s} - R^2 \rho D_c \left. \frac{\partial Y_F}{\partial r} \right|_s = \frac{\dot{m}}{4\pi}$$

- Other species

$$\frac{\dot{m}}{4\pi} Y_{i,s} - R^2 \rho D_c \left. \frac{\partial Y_i}{\partial r} \right|_s = 0$$

- Energy

$$R^2 \lambda \left. \frac{\partial T}{\partial r} \right|_s = \frac{\dot{m}}{4\pi} L + \frac{\dot{Q}_l}{4\pi} = \frac{\dot{m}}{4\pi} L_{\text{eff}}$$



Energy and Mass Transfer

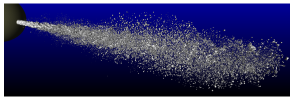
Discussion

- Three differential equation requiring six boundary conditions
- Eight boundary conditions

  ⟹ Eigenvalue problem

- Two eigenvalues: $\dot{m}$ and $\dot{Q}_l$
- Gas-phase problem can be solved if $T_{l,s}$ is known in interface condition for temperature

  ⟹ Requires analysis of liquid phase



Energy and Mass Transfer

Gas-Phase Solution

- Assuming Le = 1, define coupling function

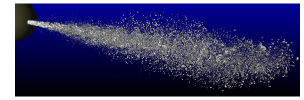
$$\beta = Y_{\mathrm{O}} - Y_{\mathrm{F}}/\nu$$

or

$$\beta = Y_{\mathrm{O}} - \frac{h}{Q\nu}$$

gives

$$L(\beta) = 0$$



Energy and Mass Transfer

- Interface flux condition
- Add

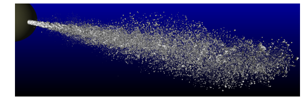
$$\frac{\dot{m}}{4\pi} Y_{O,s} - R^2 \rho D_c \left. \frac{\partial Y_O}{\partial r} \right|_s = 0$$

and

$$R^2 \rho D_c \left. \frac{\partial T / Q\nu}{\partial r} \right|_s = \frac{\dot{m}}{4\pi} \frac{L_{\text{eff}}}{Q\nu}$$

gives

$$R^2 \rho D_c \left. \frac{\partial \beta}{\partial r} \right|_s = \frac{\dot{m}}{4\pi} \left(Y_{O,s} + \frac{L_{\text{eff}}}{Q\nu} \right)$$



Energy and Mass Transfer

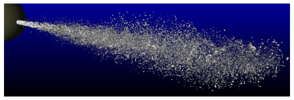
- Integration of β-equation yields

$$\frac{\dot{m}}{4\pi}\left(\beta + \frac{L_{\text{eff}} - h_s}{Q\nu}\right) - \rho D_c r^2 \frac{\partial \beta}{\partial r} = 0$$

- Second integration assuming ρD_c = const gives

$$\dot{m} = 2\pi D \rho D_c \ln \frac{\left(\beta + \frac{L_{\text{eff}} - h_s}{Q\nu}\right)_\infty}{\left(\beta + \frac{L_{\text{eff}} - h_s}{Q\nu}\right)_s}$$

Energy and Mass Transfer



- Mass flux solution

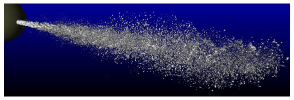
$$\dot{m} = 2\pi D \rho D_c \ln(1 + B)$$

with Spalding transfer number

$$B = \frac{Q\nu(Y_{O,\infty} - Y_{O,s}) + h_\infty - h_s}{Q\nu Y_{O,s} + L_{\text{eff}}}$$

- For fast chemistry

$$B = \frac{Q\nu Y_{O,\infty} + h_\infty - h_s}{L_{\text{eff}}}$$



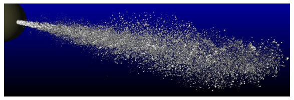
Energy and Mass Transfer

- Similar derivation for $\beta = Y_{\mathrm{O}} - Y_{\mathrm{F}}/\nu$ leading to

$$B = \frac{\nu(Y_{O,\infty} - Y_{O,s}) + Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s} + \nu Y_{O,s}}$$

- Fast chemistry

$$B = \frac{\nu Y_{O,\infty} + Y_{F,s}}{1 - Y_{F,s}}$$



Energy and Mass Transfer

Evaluate Nu number

- Heat flux

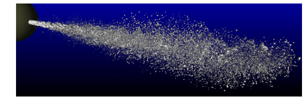
$$\dot{q}'' = \lambda \left. \frac{\partial T}{\partial r} \right|_s = k(T_{\mathrm{stoich}} - T_s)$$

- Stoichiometric temperature

$$T_{\mathrm{stoich}} - T_\infty = \frac{Q\nu}{c_p} Y_{O,\infty}$$

- Definition of Nu number

$$\mathrm{Nu} = \frac{D \left. \partial T / \partial r \right|_s}{T_\infty - T_s + \frac{Q\nu}{c_p} Y_{O,\infty}}$$



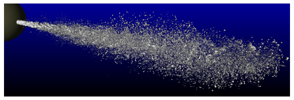
Energy and Mass Transfer

Evaluate Nu number

- Insert $\partial T / \partial r|_s$ from interface flux condition and replace L_{eff} with expression for B gives

$$\mathrm{Nu} = 2 \frac{\ln(1+B)}{B}$$

- Same expression for Sherwood number
- Caution: ΔT in the drop temperature equation has to be the same as in the definition of Nu



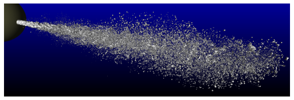
Energy and Mass Transfer

Different Levels of Approximation

- Constant droplet temperature
  - *D*²-model
- Infinite conductivity model
  - Solve droplet temperature equation

$$\frac{dT_d}{dt} = \frac{f_2 \mathrm{Nu}}{3\tau_v \mathrm{Pr}} \frac{c_{pc}}{c_{pd}} (T_\infty - T_d) + \frac{L_v}{c_{pd}} \frac{\dot{m}}{m}$$

- Liquid phase equation model
  - Solve 1D time-dependent equation for droplet
  - Too expensive



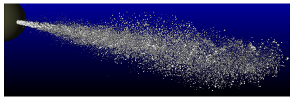
Energy and Mass Transfer

Non-Zero Reynolds Number

- Ranz-Marshall correlation (1952)

$$\dot{m} = \dot{m}_{ss}\left[1 + 0.3\mathrm{Pr}^{1/3}(2\mathrm{Re}^{1/2})\right]$$

- Correction to consider effect of convection

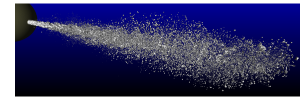


Energy and Mass Transfer

Reviews

- W. A. Sirignano: Fluid Dynamics and Transport of Droplets and Sprays

- R. S. Miller, K. Harstad, J. Bellan, Evaluation of Equilibrium and Non-Equilibrium Evaporation Models for Many-Droplet Gas-Liquid Flow Simulations, Int. J. Multiphase Flows, 24, 1025-1055, 1998



Energy and Mass Transfer

Miller et al.

$$\frac{dm}{dt} = -\frac{\mathrm{Sh}}{3\mathrm{Sc}}\frac{m}{\tau_v}H_M$$

$$\frac{dT_d}{dt} = \frac{f_2\mathrm{Nu}}{3\tau_v\mathrm{Pr}}\frac{c_{pc}}{c_{pd}}(T_\infty - T_d) + \frac{L_v}{c_{pd}}\frac{\dot m}{m}$$

Table 1
Expressions for the evaporation correction (f_2), internal temperature gradient correction ($H_{\Delta T}$) and mass transfer potential (H_M) from various models

Model	Name	f_2	$H_{\Delta T}$	H_M
M1	Classical rapid mixing†	1	0	$\ln[1 + B_{M,eq}]$
M2	Abramzon–Sirignano†	$\frac{-\dot m_d}{m_d B'_T}\left[\frac{3Pr_G\tau_d}{Nu}\right]$	0	$\ln[1 + B_{M,eq}]$
M3	Mass analogy Ia	1	0	$B_{M,eq}$
M4	Mass analogy Ib	$(1 + B_T)^{-1}$	0	$B_{M,eq}$
M5	Mass analogy IIa	1	0	$(Y_{s,eq} - Y_G)$
M6	Mass analogy IIb	$(1 + B_T)^{-1}$	0	$(Y_{s,eq} - Y_G)$
M7	Langmuir–Knudsen I	G	0	$\ln[1 + B_{M,neq}]$
M8	Langmuir–Knudsen II*	G	$\frac{2\beta}{3Pr_G}\left(\frac{\theta_1}{\tau_d}\right)\Delta_s$	$\ln[1 + B_{M,neq}]$