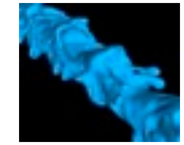


Particle Motion

Particle equation of motion

$$m\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}$$

- Assumptions
  - no evaporation/condensation or
  - spatially uniform evaporation/condensation

- Force acting on particle

$$\boldsymbol{F} = \boldsymbol{F}_b + \boldsymbol{F}_s$$

$\boldsymbol{F}_b$ ↑ body force

$\boldsymbol{F}_s$ ↑ surface force

Particle Motion

- Body Forces
  - Gravitational force

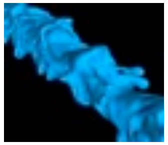
$$\boldsymbol{F}_g = m\boldsymbol{g}$$

  - Electromagnetic force

$$\boldsymbol{F}_e = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

q: particle charge; $\boldsymbol{E}$: electric field; $\boldsymbol{B}$: magnetic field

$$\Rightarrow \boxed{\boldsymbol{F}_b = m\boldsymbol{g} + q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})}$$



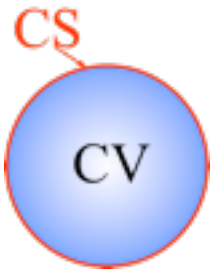
Particle Motion

- Surface Forces
  - Pressure force

$$\boldsymbol{F}_p = -\int_{cs} p_s \boldsymbol{n} dS$$

  - Shear stress force

$$\boldsymbol{F}_\tau = \int_{cs} \boldsymbol{\tau} \cdot \boldsymbol{n} dS$$



$$\Rightarrow \boxed{\boldsymbol{F}_s = \int_{cs} (-p_s \boldsymbol{n} + \boldsymbol{\tau} \cdot \boldsymbol{n}) dS}$$

Particle Motion: Drag Force

- Steady-state drag force

$$\boldsymbol{F}_D = \frac{1}{2}\rho_c C_D A|\boldsymbol{u} - \boldsymbol{v}|(\boldsymbol{u} - \boldsymbol{v})$$

  - Assumptions
    - uniform pressure field
    - no acceleration of ($\boldsymbol{u}$ - $\boldsymbol{v}$)

  - Drag coefficient C_D depends on
    - particle shape and orientation
    - Re, Ma, turbulence
    - etc.

Particle Motion: Internal Motion

- Drag coefficient
  - assume particle is non-rotating solid sphere, uniform free stream velocity, Ma << 1, and Re < 1:

$$C_D = \frac{24}{\text{Re}} \qquad \text{(Stokes flow)}$$

  - allow internal motion

$$C_D = \frac{24}{\text{Re}} \left(\frac{1 + 2/3\bar{\mu}}{1 + \bar{\mu}} \right)$$

$$\bar{\mu} = \frac{\mu_c}{\mu_d}$$

Particle Motion: Example

- Lower drag coefficient: drop or bubble?
  - water drop in air at 20C

$$\bar{\mu} = \frac{\mu_{air}}{\mu_{water}} = \frac{0.018cP}{1cP} = 0.018$$

$$\Rightarrow \quad C_D = 0.994\frac{24}{\mathrm{Re}}$$

  - air bubble in water at 20C

$$\bar{\mu} = 55.6$$

$$\Rightarrow \quad C_D = 0.673\frac{24}{\mathrm{Re}}$$

Particle Motion: Faxen Force

– allow non-uniform free stream velocity

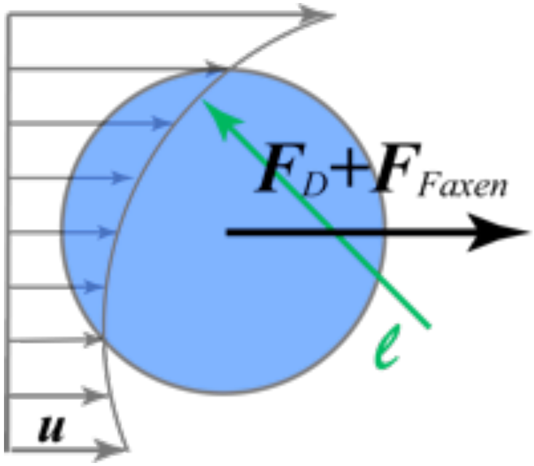
$$\boldsymbol{F}_D = 3\pi\mu_c D(\boldsymbol{u} - \boldsymbol{v}) + \mu_c \pi \frac{D^3}{8} \nabla^2 \boldsymbol{u}$$

Stokes drag — Faxen force

- When is the Faxen force important?

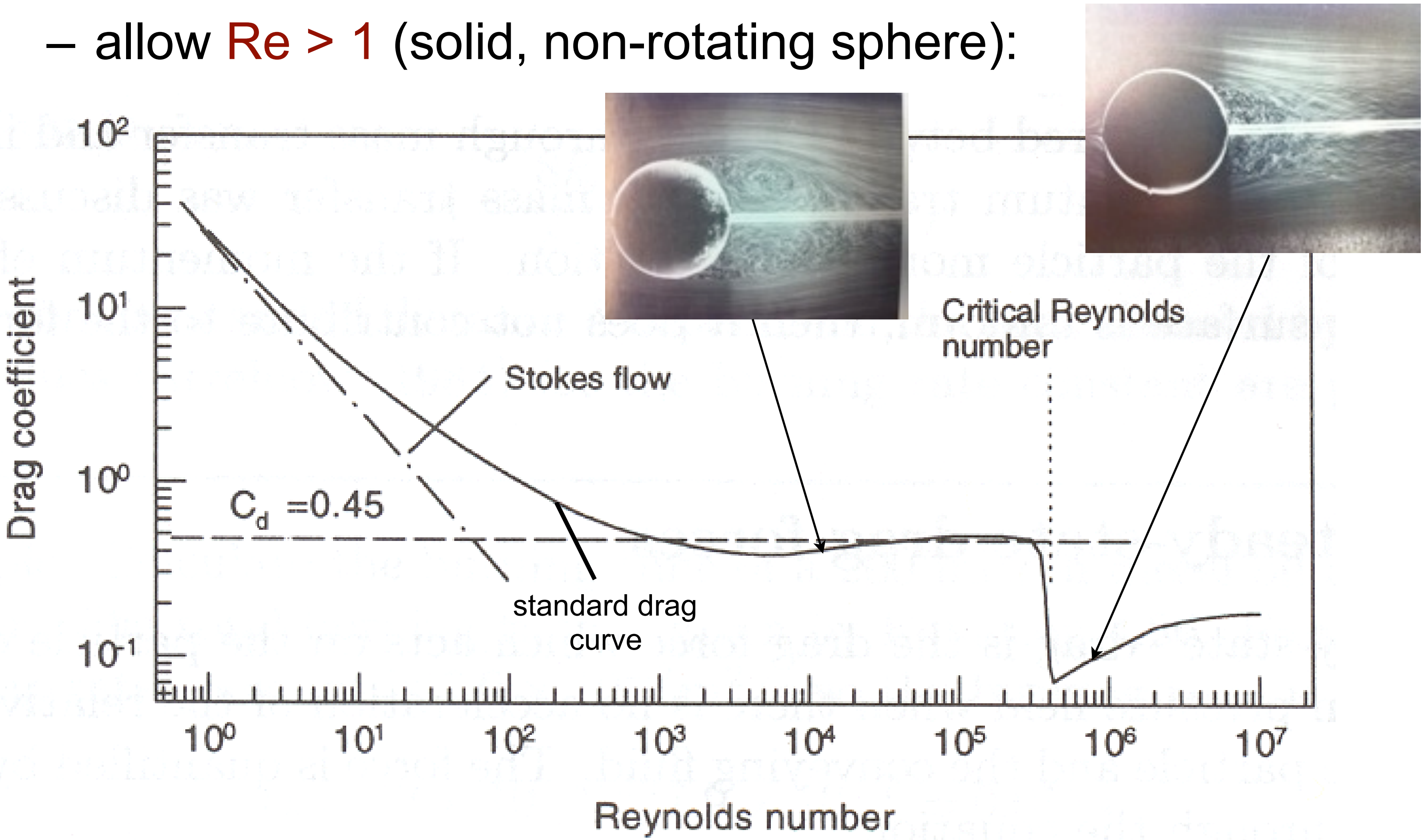
$$\frac{F_{Faxen}}{F_{Stokes}} \sim \left(\frac{D}{\ell}\right)^2$$

ℓ: characteristic length of the carrier flow



Particle Motion: Re > 1

– allow Re > 1 (solid, non-rotating sphere):



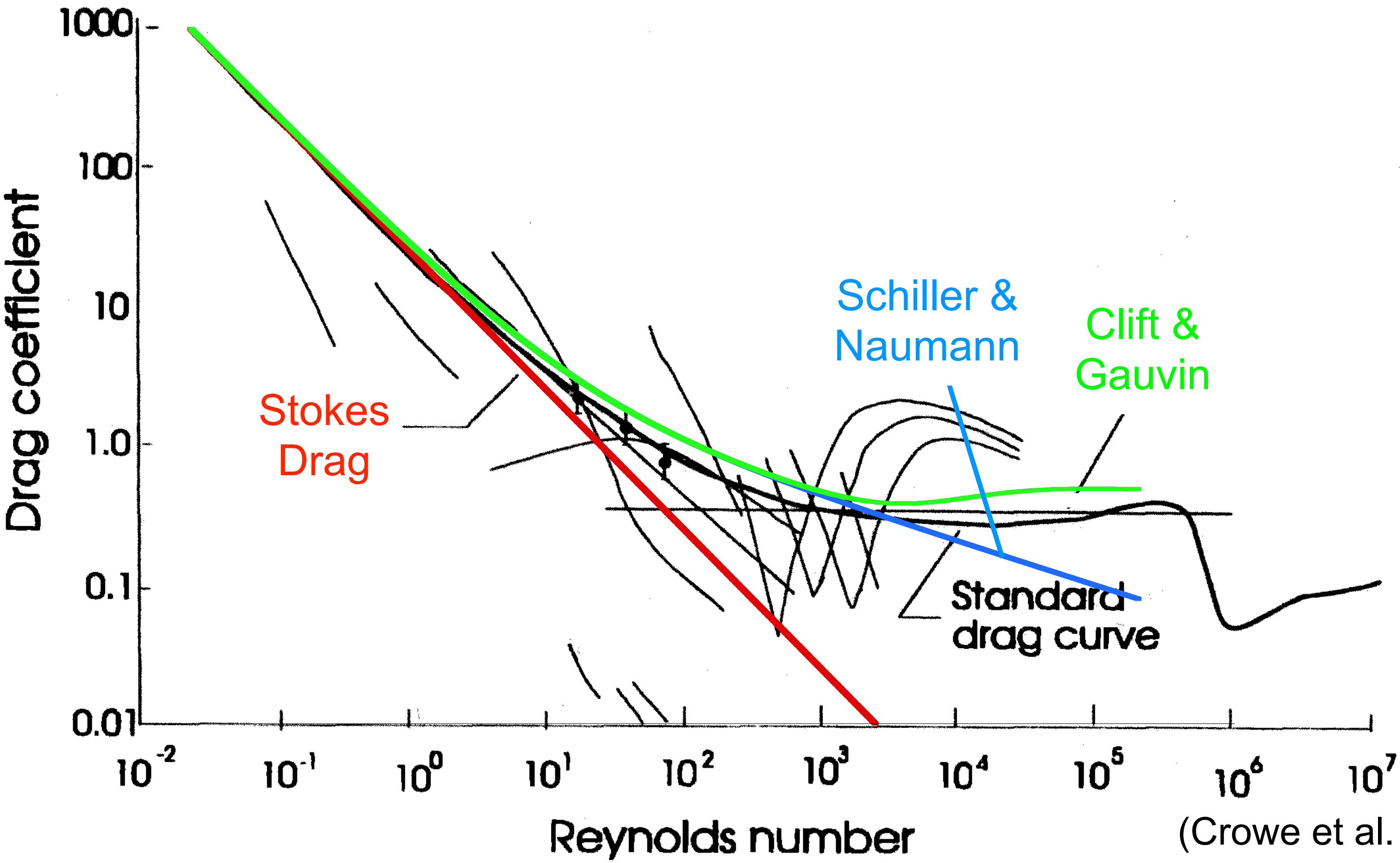
Particle Motion: Re > 1

- Drag coefficient
  - assume particle is non-rotating solid sphere in uniform free stream flow

$$C_D = f_1 \frac{24}{\mathrm{Re}}$$

- Re < 1: $f_1 = 1$ (Stokes flow)
- Re < 800: $f_1 = 1 + 0.15\mathrm{Re}^{0.687}$ (Schiller & Naumann 33)
- Re < 2x10^5: $f_1 = 1 + 0.15\mathrm{Re}^{0.687} + \frac{0.0175\mathrm{Re}}{1 + 42500\mathrm{Re}^{-1.16}}$ (Clift & Gauvin 70)

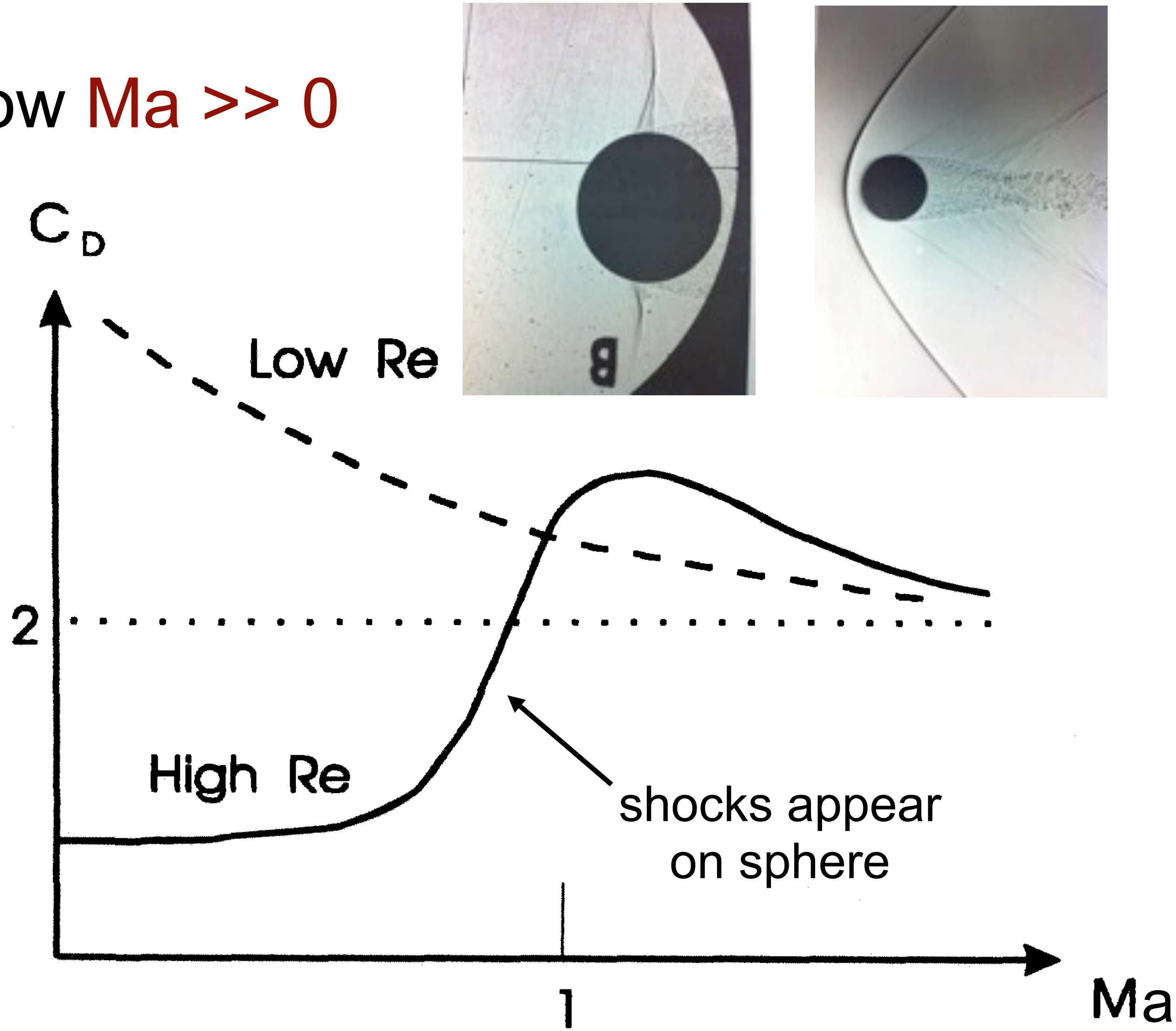
Particle Motion: Re > 1



(Crowe et al. 98)

Particle Motion: Ma >> 0

- allow Ma >> 0



(Crowe et al. 98)

Particle Motion: Drag Force

- low Re and Ma >> 0
  - Knudsen number

$$\mathrm{Kn} = \frac{\lambda}{D} \sim \frac{\mu_c}{\rho_c c D} = \frac{\mathrm{Ma}}{\mathrm{Re}}$$

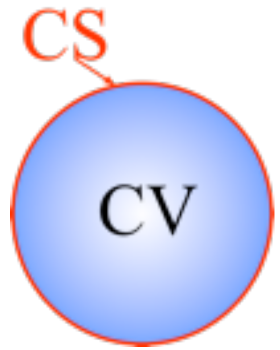
  - Kn > 1: shock wave is thicker than particle ⇒ rarefied flow
- Drag law for rarefied flows, $\mathrm{Re} < \mathrm{Re}_{crit}$:

$$C_D = 2 + (C_{D_o} - 2)e^{-3.07\sqrt{k}g(\mathrm{Re}_r)M_r/\mathrm{Re}_r} + \frac{h(M_r)}{\sqrt{k}M_r}\exp\left(-\frac{\mathrm{Re}_r}{2M_r}\right)$$

$$g(Re_r) = \frac{1 + \mathrm{Re}_r(12.278 + 0.548\mathrm{Re}_r)}{1 + 11.278\mathrm{Re}_r} \qquad h(M_r) = \frac{5.6}{1 + M_r} + 1.7\sqrt{\frac{T_d}{T_c}}$$

Particle Motion: Buoyancy

- Pressure force:



$$\boldsymbol{F}_p = -\int_{cs} p\boldsymbol{n}dS$$

$$\boldsymbol{F}_p = -\nabla p V_d = -\rho_c V_d \boldsymbol{g}$$

- Equation of motion:

$$\frac{d\boldsymbol{v}}{dt} = \frac{f_1}{\tau_V}(\boldsymbol{u} - \boldsymbol{v}) + \boldsymbol{g}\left(1 - \frac{\rho_c}{\rho_d}\right)$$

$\Rightarrow$ neglect for water drops in air

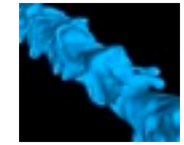
Particle Motion: Shear Stress

- Shear stress in carrier fluid:

$$\boldsymbol{F}_\tau = \int_{cs} \boldsymbol{\tau} \cdot \boldsymbol{n} dS$$

or

$$\boldsymbol{F}_\tau = \nabla \cdot \boldsymbol{\tau} V_d$$



Particle Motion: Unsteady Forces

- Allow acceleration of (***u*** - ***v***)
  - kinetic energy of fluid around a moving sphere

$$K = \frac{\pi \rho_c D^3 U^2}{24} \quad \Rightarrow \quad F_{vm} = \frac{\rho_c V_d}{2} \frac{dU}{dt}$$

  - virtual mass force

$$\boldsymbol{F}_{vm} = \frac{\rho_c V_d}{2} (\dot{\boldsymbol{u}} - \dot{\boldsymbol{v}})$$

Particle Motion: Unsteady Forces

- allow acceleration of (***u*** - ***v***)
  - Basset force: delay in boundary layer development
    - impulsively accelerated flat plate:

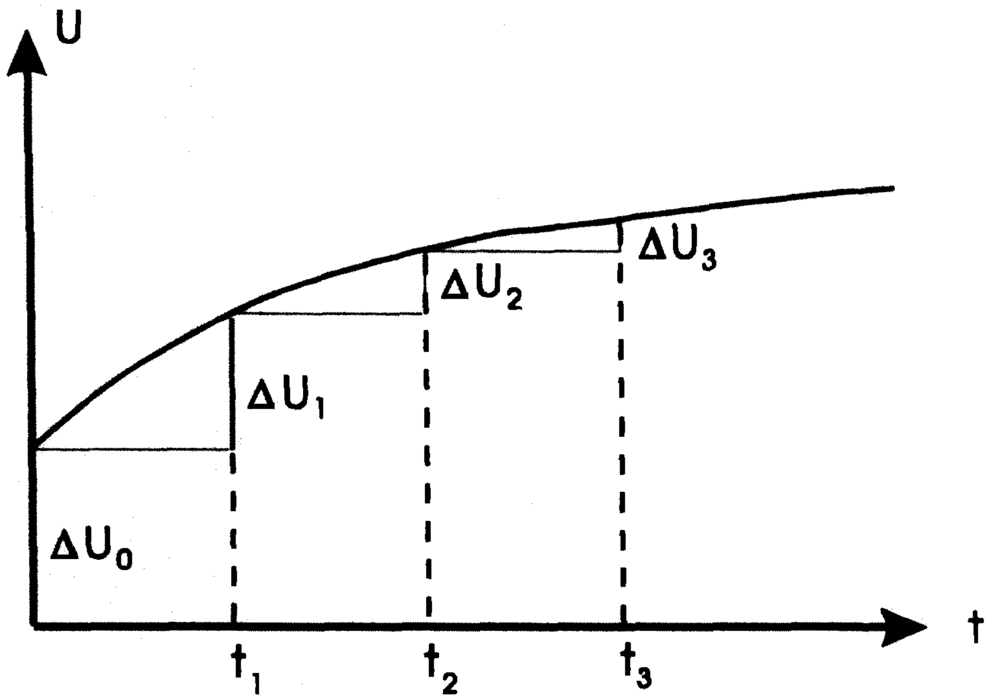
$$\tau = \frac{\sqrt{\rho_c \mu_c} u_0}{\sqrt{\pi t}}$$

    - continuously accelerated flat plate:

$$\tau = \sqrt{\frac{\rho_c \mu_c}{\pi}} \int_0^t \frac{du/dt'}{\sqrt{t - t'}} dt'$$

Particle Motion: Unsteady Forces

Stepwise impulsive accelerated flat plate



(Crowe et al. 98)

Particle Motion: Unsteady Forces

- allow acceleration of (***u*** - ***v***)
  - Basset force:

$$\boldsymbol{F}_B = \frac{3}{2} D^2 \sqrt{\pi \rho_c \mu_c} \int_0^t \frac{\dot{\boldsymbol{u}} - \dot{\boldsymbol{v}}}{\sqrt{t - t'}} dt'$$

with initial velocity:

$$\boldsymbol{F}_B = \frac{3}{2} D^2 \sqrt{\pi \rho_c \mu_c} \left[\int_0^t \frac{\dot{\boldsymbol{u}} - \dot{\boldsymbol{v}}}{\sqrt{t - t'}} dt' + \frac{(\boldsymbol{u} - \boldsymbol{v})_0}{\sqrt{t}} \right]$$

Basset-Boussinesq-Oseen Eq.

- steady state drag + pressure force + shear stress force + unsteady forces + body forces
- neglect: non-uniformity of velocity field, internal motion, Mach effects, non-sphericity, etc...

$$
\begin{aligned}
m\frac{d\boldsymbol{v}}{dt} = {} & 3\pi f_1 \mu_c D(\boldsymbol{u}-\boldsymbol{v}) \\
& + V_d\left(-\nabla p + \nabla\cdot\boldsymbol{\tau}\right) \\
& + \frac{\rho_c V_d}{2}\left(\dot{\boldsymbol{u}}-\dot{\boldsymbol{v}}\right) \\
& + \frac{3}{2}D^2\sqrt{\pi\rho_c\mu_c}\left[\int_0^t \frac{\dot{\boldsymbol{u}}-\dot{\boldsymbol{v}}}{\sqrt{t-t'}}dt' + \frac{(\boldsymbol{u}-\boldsymbol{v})_0}{\sqrt{t}}\right] \\
& + m\boldsymbol{g}
\end{aligned}
$$

Basset-Boussinesq-Oseen Eq.

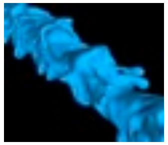
- carrier phase Navier-Stokes equation

$$-\nabla p + \nabla \cdot \boldsymbol{\tau} = \rho_c \dot{\boldsymbol{u}} - \rho_c \boldsymbol{g}$$

- isolated particle BBO

$$\left(1 + \frac{1}{2}\frac{\rho_c}{\rho_d}\right)\frac{d\boldsymbol{v}}{dt} = \frac{f_1}{\tau_V}(\boldsymbol{u} - \boldsymbol{v}) + \frac{3}{2}\frac{\rho_c}{\rho_d}\dot{\boldsymbol{u}} + \boldsymbol{g}\left(1 - \frac{\rho_c}{\rho_d}\right)$$
$$+\sqrt{\frac{9}{2\pi}\frac{\rho_c}{\rho_d}\frac{1}{\tau_V}}\left[\int_0^t \frac{\dot{\boldsymbol{u}} - \dot{\boldsymbol{v}}}{\sqrt{t - t'}}dt' + \frac{(\boldsymbol{u} - \boldsymbol{v})_0}{\sqrt{t}}\right]$$

for $\dfrac{\rho_c}{\rho_d} << 1 \quad \Rightarrow \quad \dfrac{d\boldsymbol{v}}{dt} = \dfrac{f_1}{\tau_V}(\boldsymbol{u} - \boldsymbol{v}) + \boldsymbol{g}$



Particle Motion: Non-Spheres

- Shape factor

$$\psi = \frac{A_s}{A}$$

A_s: surface area of sphere with same V

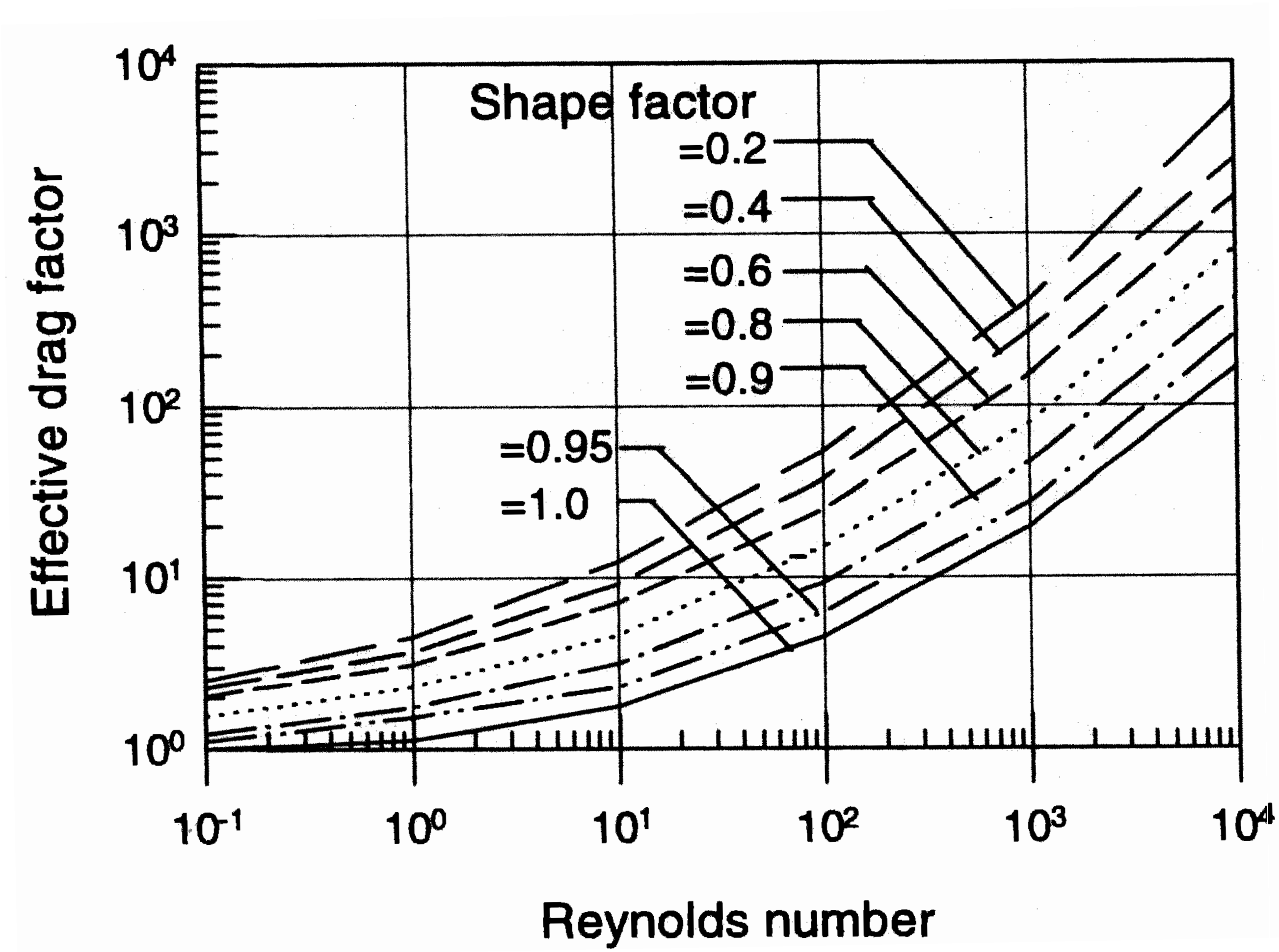
A: actual surface area

- effective drag factor

$$\frac{d\boldsymbol{v}}{dt} = \frac{f_1}{K^2}\frac{1}{\tau_V}(\boldsymbol{u}-\boldsymbol{v}) + \boldsymbol{g} = \frac{f_e}{\tau_V}(\boldsymbol{u}-\boldsymbol{v}) + \boldsymbol{g}$$

Particle Motion: Non-Spheres

- Effective drag factor



(Crowe et al. 98)

Particle Motion: Blowing Effect

- Stefan flow due to evaporation reduces C_D

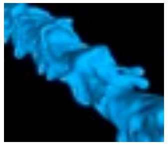
$$C_D = \frac{C_{D,0}}{1 + B}$$

(Eisenklam 67)

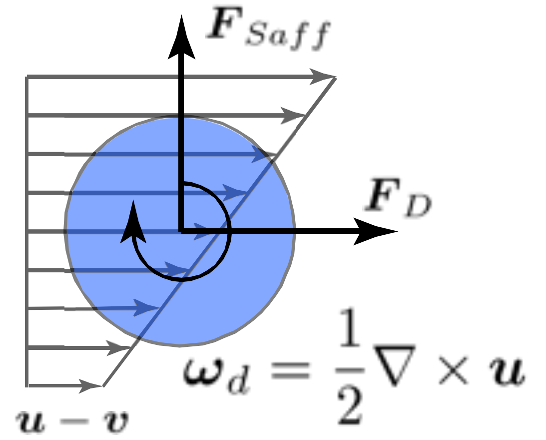
$C_{D,0}$: without Stefan flow

B: Spalding transfer number

- important for particle combustion ($1 < B < 10$)



Particle Motion: Lift Forces



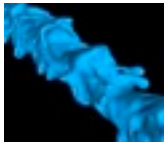
- Saffman force
  - Rotation by velocity gradient

$$\mathrm{Re}_G = \frac{D^2}{\nu_c}\frac{du}{dy}$$

$$\boldsymbol{F}_{Saff} = 1.61\mu_c D|\boldsymbol{u} - \boldsymbol{v}|\sqrt{\mathrm{Re}_G}$$

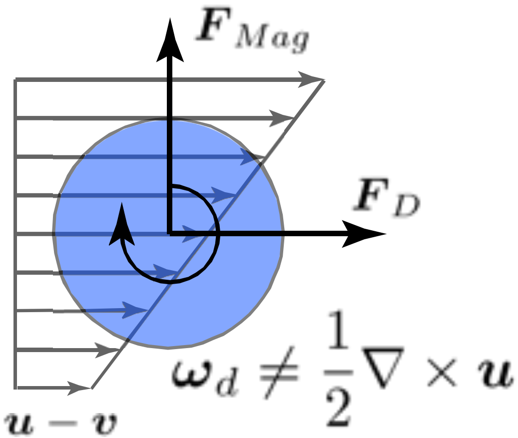
  - Assumptions:

$$\mathrm{Re} << 1 \qquad \mathrm{Re}_G << 1 \qquad \mathrm{Re} << \sqrt{\mathrm{Re}_G}$$



Particle Motion: Lift Forces

- Magnus force



  – Rotation by interactions with particles or walls

$$\boldsymbol{F}_{Mag} = \frac{\pi}{8} D^3 \rho_c \left[\left(\frac{1}{2} \nabla \times \boldsymbol{u} - \boldsymbol{\omega}_d \right) \times (\boldsymbol{u} - \boldsymbol{v}) \right]$$

  – Assumptions:

$$\mathrm{Re} = O(1)$$

Particle Motion: Torque

- Shear stress distribution on particle surface induces a torque

  - low Re:

$$\boldsymbol{T} = \pi \mu_c D^3 \left(\frac{1}{2} \nabla \times \boldsymbol{u} - \boldsymbol{\omega}_d \right)$$

(Stokes flow)

  - 20 < Re < 1000:

(Dennis et al. 80)

$$\boldsymbol{T} = -2.01 \mu_c D^3 \omega_d \left(1 + 0.201 \sqrt{\mathrm{Re}_d} \right)$$

with $\mathrm{Re}_d = \dfrac{\rho_c \omega_d D^2}{4 \mu_c}$

Particle Motion

- allow more than one particle:

  ⇒ particle clouds, fluidized beds, sedimentation

  – drag force is function of
    - particle spacing
    - particle size distribution