Problem Set #1

Due March 3, 2010

Problem 1 – Isotropic Decaying Turbulence

In Direct Numerical Simulations (DNS), the Navier-Stokes equations are solved directly without any modeling assumptions. Results from DNS are often used to study turbulence, because of the full description of the velocity field and the velocity statistics obtained from such simulations. Isotropic decaying turbulence is a particularly simple turbulence scenario, for which all turbulence statistics are independent of the spatial location and even of the spatial orientation. Throughout this turbulence course, we will use a turbulence field from a DNS of isotropic turbulence to get some experience about how real turbulence looks. The first step for you is to download the data set from `http://www.colorado.edu/MCEN/cmes/courses/turbulence.html`. It includes the velocity field from the simulation at a given time (HIT_field_sp.dat) as well as a Matlab script for reading the file (read_box.m). The simulation was done for a box of dimensions $2\pi \times 2\pi \times 2\pi$, using $256 \times 256 \times 256$ grid points. Periodic boundary conditions were used in all three directions. The data file contains the three components of velocity as well as the pressure.

(a) Using these data, plot a 1D profile of velocity vs. coordinate along a line. You are free to choose any line you want (for e.g. (i, j = 10, k = 10)). By looking at the plot, visually estimate:

   i. The magnitude of the largest velocity fluctuations and the length over which these fluctuations occur.

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(b) Compute the mean and variance of the three velocity components.

(c) Compute and plot the PDF of a velocity component. In isotropic turbulence, all three velocity components have the same statistics. So you can use all three components to get better statistics. Decribe the PDF by computing the mean, variance, skewness and kurtosis. Discuss these values in comparison to the normal distribution. Can you estimate a length scale of the turbulence from the PDF?

(d) Discuss the fundamental properties of the Reynolds stress tensor using arguments about the symmetry of the flow. What can you say about the values on and off the diagonal?

(e) Evaluate numerically all six components of the Reynolds stress tensor by averaging over the entire field. Compare your results to the theory (question (d)). Keep in mind that the data field might not be perfect.

Tip: Do not wait until the last moment to start on this problem. The data set is big (about 250 MB). A computer with 1GB or more of RAM is recommended, but at least 512 MB of RAM are required. We will be using this data set in future homeworks as well, so *make sure you understand what the data set represents. If you dont have a computer that can handle the data set, let me know, and I will provide you with access to a computer lab. If you have any trouble using the data set, you should talk to me immediately.*

Problem 2 – Mixing Length Rationale for a Scalar

Passive scalars (for example contaminants, temperature, etc...) are transported by turbulent motion similarly to the momentum. Consider the mass fraction $\phi(\boldsymbol{x})$ of a passive scalar with mean $\langle \phi \rangle$ and fluctuation ϕ' that is in a turbulent field with mean velocity $\langle \boldsymbol{U} \rangle$ and velocity fluctuation $\boldsymbol{u}$. Using the mixing length argument, derive an eddy-diffusion formula for $\langle \boldsymbol{u}\phi' \rangle$ analogous to the one for the eddy viscosity.