Problem Set #2

Due March 17, 2010

Problem 1 – Mixing Layer

For this homework, we will use a 3D data field corresponding to a mixing layer. The first step for you is to download the data set from `http://www.colorado.edu/MCEN/cmes/courses/turbulence.html`. It includes the velocity field at a given time and a Matlab script for reading the file. The simulation was done for a box of dimension $[0,4]\times[-1,1]\times[0,1]$, using $512\times256\times128$ grid points. Periodic boundary conditions were used for the streamwise (x) and spanwise (z) directions. The upper part of the domain ($y>0$) was initialized with a uniform velocity $U=+1$ while the lower part ($y<0$) was initialized with $U=-1$.

1. Using these data, compute the mean streamwise velocity profile by averaging over a plane at constant height. Plot it vs the y coordinate.
$$\bar{U}(y) = \langle U(x,y,z)\rangle$$
2. From the mean velocity field, compute the mean vertical gradient of the streamwise velocity
$$\frac{d\bar{U}}{dy}(y)$$
3. Discuss the fundamental properties of the Reynolds stress tensor using arguments about the symmetry of the flow. What is the main difference between this tensor and that of homogeneous isotropic turbulence?
4. Evaluate numerically and plot the four components of the Reynolds stress tensor (including $\langle uv\rangle$) by averaging over a plane at constant height. What can you say about the variances of u,v, and w?
5. Estimate the mixing length scale from the Reynolds stress (question d) and the velocity gradient (question b). Assume it is a constant throughout the domain. Plot the $\langle uv\rangle$ correlation predicted by the mixing length model and the exact values (question d).

Problem 2 – Relative magnitude of Turbulent Reynolds Number

Determine the ratio of the jet Reynolds number and the turbulent Reynolds number, which is defined as
$$\mathrm{Re}_t = \frac{u'l}{\nu} = \frac{\nu_t}{\nu}\,.$$
Estimate the value for the normalized eddy viscosity from Fig. 5.10 in Pope's Turbulent Flows book. Take B and S from Table 5.1 in the same book.

Round the value you obtain up or down and memorize it for the rest of your life!

Problem 3 – Scalar Transport

Let $\phi(\boldsymbol{x}, t)$ be a conserved scalar quantity with conservation equation

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\boldsymbol{U}\phi) = \Gamma \nabla^2 \phi \,.$$

The Reynolds averaged equation for this scalar is given by

$$\frac{\partial \langle \phi \rangle}{\partial t} + \langle \boldsymbol{U} \rangle \cdot \nabla \langle \phi \rangle = \Gamma \nabla^2 \langle \phi \rangle - \nabla \cdot (\langle \boldsymbol{u}\phi' \rangle) \,.$$

1. Show that the transport equation for the scalar variance is

$$\frac{\partial \langle \phi'^2 \rangle}{\partial t} + \langle \boldsymbol{U} \rangle \cdot \nabla \left\langle \phi'^2 \right\rangle = \Gamma \nabla^2 \left\langle \phi'^2 \right\rangle - \nabla \cdot (\left\langle \boldsymbol{u}\phi'^2 \right\rangle) - 2 \langle \boldsymbol{u}\phi' \rangle \cdot \nabla \langle \phi \rangle - 2\Gamma \langle \nabla \phi' \cdot \nabla \phi' \rangle$$

2. Identify the production term, the dissipation term, the turbulent scalar flux term, and the viscous transport term.

3. Introduce the gradient-diffusion model (Eq. 4.42 in Pope's Turbulent Flows book) into the scalar variance equation.

4. For part e) of this problem you need a model for ε_ϕ. Derive this by assuming that there is a balance between production and dissipation in the scalar variance equation, and that the eddy diffusivity is a known quantity.

5. In the k-ε model, transport equations are solved for k and ε, so that these are known quantities. Assuming that the time scale of the turbulence decay k/ε is equal to the time scale of the scalar variance decay defined analogously, and using the model for ε_ϕ, provide a model for the scalar variance.