Problem Set #3

Due April 14, 2010

Back to Isotropic Decaying Turbulence

In the first homework, you visually estimated the length scale of the large velocity fluctuations, and you evaluated the PDF of the velocity, from which no length scales can be estimated. In this homework, you will compute the several length scales and examine spectral information.

1. Using the first component of velocity (or again all three since the flow is isotropic) compute and plot the two-point correlation function $R_{11}(r)$. Discuss the resulting curve.

$$R_{11}(r) = \langle u_1(\boldsymbol{x}) u_1(\boldsymbol{x} + r\boldsymbol{e}_1) \rangle$$

2. Evaluate the integral length scale L_{11}. Compare this value to the visualy estimated value from the first homework.

3. Compute the longitudinal Taylor microscale λ_f using the following relation

$$\frac{2u'^2}{\lambda_f^2} = \left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle,$$

   where u'^2 is the variance of a single velocity component.

4. Using any one plane from the data (for eg. (i, j, k=10)), plot contours of a velocity component. On your contour plot, mark the size of the largest structure you see. How does this compare with the Taylor microscale λ_f and the integral length scale L_{11}?

5. Compute and plot the one-dimensional energy spectrum function $E_{11}(\kappa_1)$. Mark the various regions of the energy cascade on the plot. Also plot the compensated spectrum. Do you see an inertial subrange? Estimate the slope of your plot as $\kappa_1 \longrightarrow 0$. Explain your result.

6. Using the DNS data, compute and plot the 1-D dissipation spectrum

$$D_1(\kappa_1) = 2\nu\kappa_1^2 E_{11}(\kappa_1).$$

   The dynamic viscosity is $\nu = 0.01$. Also on the same figure, plot the cumulative dissipation

$$\varepsilon_{(0,\kappa_1)} \equiv \int_0^{\kappa_1} D_1(\kappa_1^{'}) d\kappa_1^{'}.$$

   Discuss the results.

7. Evaluate L_{DI} and L_s from the graphs. Compare these values to the values derived in class.

Kolmogorov scaling

Let ϕ be a passive scalar quantity, such as a contaminant or a small temperature perturbation. Consider a small patch of the contaminant in a turbulent flow. The patch initially has a size l, is significantly larger than the Kolmogorov length, but small compared with the integral length scale.

1. What is the growth rate of the patch per unit time (dl/dt)?
2. What is the decay of the variance of the scalar $\langle \phi'^2 \rangle$ per unit time?

Tip: This problem is about Kolmogorov scaling. You have to think about what causes the mixing, what eddies are involved, make simplifying assumptions, and use scaling arguments.