Problem Set #4

Due April 28, 2010

Turbulent pipe flow

For this homework, we will use a 3D data field corresponding to a turbulent pipe flow. The first step for you is to download the data set from `http://www.colorado.edu/MCEN/cmes/courses/turbulence.html`. It includes the velocity field at a given time, a Matlab script for reading the file, and a file corresponding to the location of the gridpoints in the radial direction. The simulation was done for a cylinder of dimension $L_x \times L_r \times L_\theta = 10R \times R \times 2\pi$ (with $R = 1$), using $256\times96\times128$ grid points. Periodic boundary conditions were used for the streamwise (x) and spanwise (θ) directions. The wall is located at $r = R$ and the centerline corresponds to $r = 0$. The Reynolds number used for the simulation is $\mathrm{Re}_\tau = \frac{u_\tau R}{\nu} = 180$ and the viscosity is $\nu = 3.7665 \times 10^{-4}$.

1. Using these data, compute the mean streamwise velocity profile by averaging over a plane at constant radius. Plot the normalized velocity (u^+) vs the normalized distance to the wall (y^+).
$$\bar{U}(y) = \langle U(x, r, \theta)\rangle$$
2. From the mean velocity field, compute the two parameters (κ and B) for the log law. Compare these values to those for a turbulent channel flow.
3. On the same plot, plot the relation for the log law. Also plot the relation for the viscous sublayer.

Turbulent channel flow

Reynolds numbers

For a turbulent channel flow:

1. Derive a relation between Re_0, u^+, and Re_τ.
2. From the log-law, estimate the centerline value of u^+.
3. From these results, provide a relation between Re_0 and Re_τ.

Kolmogorov scale

In the log-law region, we saw that $du^+/dy^+ = 1/\kappa y^+$, and also that production and dissipation are almost in balance. Using these two ideas, obtain the following estimates for the Kolmogorov scale in the log-law region:

$$\frac{\eta}{\delta_\nu} = (\kappa y^+)^{1/4}, \quad \frac{\eta}{L} = \frac{1}{C_L}\left(\frac{\kappa}{y^{+3}}\right)^{1/4}.$$

In the second relation, L is the characteristic large scale of the turbulent channel flow, $L = k^{3/2}/\epsilon$. In the log layer, it can be shown that $L = C_L y$, with $C_L \approx 2.5$.